**1**

7th February, 2021

Question 1 -

a) 'A' belongs to 2nd Group, 3rd Period
∴ 'A' is Magnesium (Mg)

b) 'B' is the most electronegative element
∴ 'B' is Flourine (F)

Molecular Formula of compound $A_x B_y$ = $\boxed{MgF_2}$

Molar mass = $1 \times 24 + 2 \times 19$
= $24 + 38$
= $\boxed{62}$

**2**

MPI - Entrance Exam - Feb 7, 2021
chemistry

2. Write sum of atomic numbers of the following elements :

i) belongs to $17^{th}$ group and $4^{th}$ period in modern periodic table.
→ Bromine (Br)
Atomic number = $2+8+8+18-1$
= 35

ii) Largest element of $4^{th}$ period
→ Atomic size decreases across a period ⇒ Largest = Potassium (K)
Atomic number = 19
⇒ Sum of atomic numbers = $19 + 35$
= $\boxed{54}$

~ Solution by Aryan Raval
10 - oly

**3**

7th February 2021

3. $Cu$ + dilute $HNO_{3(aq)} \longrightarrow Cu(NO_3)_{2(aq)} + H_2O_{(l)} + X_{(g)}$

$Cu$ is below H in reactivity series

∴ It produces NO

∴ X = NO

N = 14 , O = 16

∴ Molar Mass of X = 14+16 = 30 g/mol

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**4**

7 Feb 2021

Q.4) $\overset{+4}{MnO_2} + 2H\overset{-1}{Cl} \longrightarrow \overset{+2}{Mn}Cl_2 + \overset{0}{Cl_2} + H_2O$

(2) (2)

4 - Cl   2 - O

∴ $MnO_2 + 4HCl \longrightarrow MnCl_2 + Cl_2 + 2H_2O$

is balanced reaction

12 moles $MnCl_2$ requires 48 mole HCl

→ Avadhoot Nikam

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**5**

7 Feb 2021

5.

$\rightarrow$ $2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2$

$\overset{(+7)}{KMnO_4}$ is reduced to $\overset{(+2)}{MnSO_4}$
Hence, it is the oxidizing agent

$FeSO_4$ is oxidised to $Fe_2(SO_4)_3$
Hence, it is the reducing agent

$\therefore$ $KMnO_4$ = OA , $FeSO_4$ = RA

MM of $KMnO_4$ = 39 + 55 + 4 × 16 = 158 , MM of $FeSO_4$ = 56 + 32 + 4 × 16 = 152

Difference in molar mass = 158 - 152 = 6

$\boxed{\text{Ans: } 6}$

Solution by - Sahil Khose

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**6**

Q6.

Ans: The Complete reaction is : $Fe(s) + H_2O(g) \xrightarrow{heat} Fe_3O_4(s) + H_2(g)$

$\therefore$ 'A' is $Fe_3O_4$

Total atoms in one molecule of $Fe_3O_4$ = 3+4 = $\boxed{7}$ ★

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**7**

February 7, 2021

Chemistry

Q7. (1) X, Y, Z ∈ 3rd period and are silvery white
$\Rightarrow$ These are metals. (Na, Mg, Al)

(2) Z reacts rapidly & vigourosly with water + $H_2 \uparrow$
$\Rightarrow$ Z is Na.

(3) Y can't displace X
$\Rightarrow$ Y = Al and X = Mg.

(4) Z can displace Y from it's aq. solution.
Z = Na, Y = Al

Check: $Na + Al_2(SO_4)_3 \rightarrow Na_2SO_4 + Al$.

$\Rightarrow$ Y = Al is the least reactive of all X, Y, Z.
Atomic Mass = $\boxed{27}$

Solution by Shubhra N.

**8**

DATE / /

07-Feb-21 Chemistry Q. No. 8

1st compound in alkanne series:- $CH_4$ MM - 16
1st compound in alkenes series:- $C_2H_4$ MM - 28
1st compound in alkyne series:- $C_2H_2$ MM - 26
$16 + 28 + 26 = \boxed{70}$

- Chinmay Khairnar

**9**

7 February 2021 Chemistry

Q.9) A Hydrocarbon $C_xH_y$ on complete combustion produces a gas 'B' and water. If the molar mass of gas 'B' and Hydrocarbon $(C_xH_y)$ are same, then value of $(x+y)$ is __.

Ans When Hydrocarbon is burnt, $CO_2$ gas is produced with water.

$\therefore$ Gas B $\rightarrow$ $CO_2$ gas.

$\therefore$ MM of B = MM of $CO_2 = 12 + (16\times2) = 44$.

$\therefore$ According to given condition,

MM of $C_xH_y$ = MM of $CO_2$

$\therefore$ MM of $C_xH_y = 44$

This hydrocarbon can be alkane, alkene or alkyne.

Case-1 :- $C_xH_y$ is alkane.

$\therefore x = n, \; y = 2n+2$.

$\therefore$ MM of $C_nH_{2n+2} = 12n + 2n + 2$

$44 = 14n + 2$

$\therefore 14n = 42$

$\therefore n = 3$

$\therefore x = 3 \quad y = 2(3)+2 = 8$ Possible ✓.

Hydrocarbon will be $C_3H_8$. ~~(ethane)~~

Case-2 :- $C_xH_y$ is alkene.

$\therefore x = n \quad y = 2n$

$\therefore$ MM of $C_nH_{2n} = 12n + 2n$

$44 = 14n$

$\therefore n = x = \frac{44}{14}$ ...(Not possible as $x$ cannot be fraction).

**10**

Feb, 7, 2021

methyl ethanoate → $CH_3COOCH_3$

methanol → $CH_3OH$

sulphuric acid → $H_2SO_4$

acetic acid → $CH_3COOH$

$$CH_3COOH + CH_3OH \xrightarrow{H_2SO_4} CH_3COOCH_3 + H_2O$$

0.25 mol

$$CH_3COOH \longrightarrow CH_3COOCH_3$$

? ⟶ 0.25 mol

0.25 mol

mm of acetic acid = 60

mass = $0.25 \times 60 = \underline{15 gm}$

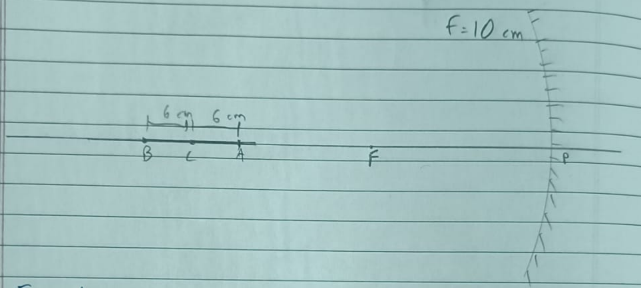
— Sanvi Deokar

**11**

7 Feb Physics

Q11

Ans




For A:

AP = 20 − 6 = 14

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\therefore \frac{1}{-10} = \frac{1}{-14} + \frac{1}{v_1}$$

$$\therefore \frac{1}{v_1} = \frac{1}{14} - \frac{1}{10}$$

$$= \frac{5-7}{70}$$

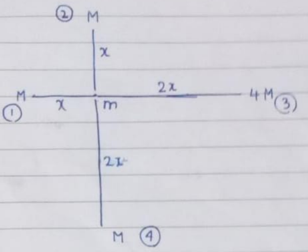
$$\therefore v_1 = -35 \text{ cm}$$

For B:

BP = 26 cm

**12**

12




USING $F = \frac{GMm}{r^2}$

Horizontal forces on 'm' due to ① & ③

due to ① $\Rightarrow \frac{GMm}{x^2}$ (rightwards)

due to ③ $\Rightarrow \frac{G4Mm}{4x^2}$ (leftwards)

$\Rightarrow$ horizontally $F_{net} = 0$ — $\bar{A}$

Vertically : force due to ② & ④

due to ② $\Rightarrow \frac{GMm}{x^2}$ (downwards)

due to ④ $\Rightarrow \frac{GMm}{4x^2}$ (upwards)

$\Rightarrow$ vertically $F_{net} = \frac{3GMm}{4x^2}$ (downwards) — $\bar{B}$

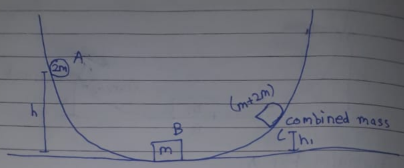
Thus $F_{net}$ on 'm' $= \bar{A} + \bar{B} = \frac{3GMm}{4x^2}$

$\Rightarrow k = 3; l = 4 \Rightarrow k + l = \boxed{7}$

~ SOLUTION BY
ABHA SHELKE
X OLYMPIAD 22-23

**13**

13




Initially at A,

KE = 0, PE = mgh $\Rightarrow$ 2mgh. — ①

When ball reaches B,

$KE = \frac{1}{2} m_{ball} v^2 = \frac{1}{2}(2m)v^2$, PE = 0. — ②

$\therefore$ By Conservation of Mechanical Energy (COME) Principle,

Initial Energy = Final Energy.

$\therefore 2mgh = \frac{1}{2}(2m)v^2$

$\therefore \boxed{v^2 = 2gh}$

$\therefore \boxed{v = \sqrt{2gh}}$

Now, at B, Initial velocity of ball $= \sqrt{2gh}$,

Initial velocity of block = 0.

$\therefore$ By conservation of Linear Momentum,

$m_1u_1 + m_2u_2 = (m_1 + m_2) v_{combined\ mass}$

$\therefore (2m)\sqrt{2gh} + 0 = (3m) v_{combined\ mass}$

$\therefore 2\cancel{m}\sqrt{2gh} = 3\cancel{m}\, v_{combined\ mass}$

$\therefore v_{combined\ mass} = \frac{2}{3}\sqrt{2gh}$. — ③

Now, For combined mass,

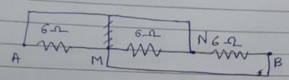
**14**

Phy
30/11/22

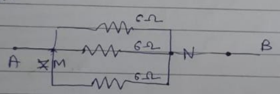
Feb 7 2021 paper
IIT Jee Training batch 2021-2023

Physics

Q14. Find Reffective




These 3 resistances can also be written as




We can rewrite the resistors as they are in parallel

$\therefore$ Req $= \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$\frac{1}{R_P} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$\frac{1}{R_P} = \frac{3}{6}$

$\boxed{R_P = 2\ \Omega}$

If you replace the resistances with a single resistance of $2\Omega$, the current will remain same

$\Rightarrow$ equivalent resistance $= 2\Omega$

Solⁿ by: Mugdha Joshi :)
X Oly batch

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**15**

Q15 7 Feb 2021

CASE I: Effective resistance $= 3R + R = 4R$

$\therefore P_1 = \frac{(\text{Voltage})^2}{\text{Resistance}}$

$= \frac{V^2}{4R}$

CASE II: $(\text{Effective resistance})^{-1} = \frac{1}{3R} + \frac{1}{R}$

$= \frac{4}{3R}$

$\therefore$ Effective resistance $= \frac{3R}{4}$

$\therefore P_2 = \frac{4V^2}{3R}$

$\frac{P_1}{P_2} = \frac{V^2}{4R} \times \frac{3R}{4V^2}$

$= \frac{3}{16}$

$\therefore m = 3 \quad n = 16$

$\therefore m + n = 19$

By: Sayali Gaikwad
X Olympiad - 22

**16**

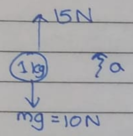
* 7-Feb-2021 (Physics) ⇒

Q.16 A ball of mass 1 kg is at rest on the ground. An upward force of 15 N is applied on it for 2s after which it moves freely under gravity. Find the max. height (in meters) from the ground to which the ball will rise:

→

- For first 2 seconds (when the force is applied) ⇒

FBD of ball ⇒




$\therefore 15N - 10N = ma$

$5N = 1 \times a$

$\therefore a = 5 m/s^2$

- For first 2 seconds (motion of the ball) ⇒

$u = 0,\ a = 5m/s^2,\ t = 2\ sec$

$\therefore s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = 0 + \frac{1}{2} \times 5 \times 2 \times 2$

$\therefore s_1 = 10m$

$\therefore v = u + at \Rightarrow v = 0 + 5 \times 2$

$\therefore v = 10m/s$

- For the ball after the force is removed, it is under free fall with $v = 10m/s$ (upward).

$\therefore$ initial vel. $= v = 10m/s,\ a = -g = -10m/s^2,\ v_f = 0$

final velocity at topmost point

$\therefore v^2 = u^2 + 2as \Rightarrow 0^2 = 10^2 + 2(-10)s_2$

$\therefore 20s_2 = 100$

$\therefore s_2 = 5m$

$\therefore$ Max. height reached by the ball $= s_1 + s_2$

$= 10 + 5$

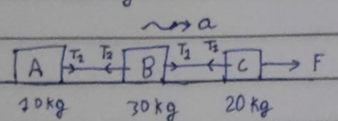
$= \boxed{15m}$

-Solution by Aditi Nimbolkar.

**17**

7 Feb 2021 Physics

17)




$m_{A,B,C} = 60kg$

$a_A = a_B = a_C = a = \frac{F}{60}$

$FBD_C$ gives $F - T_1 = m_C a$

$FBD_{A,B}$ gives $T_1 = m_{A,B} a$

$FBD_A$ gives $T_2 = m_A a$ $\quad T_1 \geq T_2$

$T_1 = T_{max} = 480 = 40a \geq T_2$

$a = 12 \frac{m}{s^2} = \frac{F}{60}$ $\quad F = 720N$

**18**

$\boxed{90kg}$ height $= 40 \times 20 = 800cm$

$= 8m$

time $= 80 secs$

$\therefore PE = mgh$

$= 900 \times 80 (5 \times 8)$

$= \boxed{72000\ W}$

$= \boxed{72kW}$

**19**

$1^{st}$ term $= a$

common difference $= d$

$$S_9 = \frac{9}{2}(2a + 8d) = 28$$

$$9(a + 4d) = 28$$

$$a + 4d = \frac{28}{9} \quad - ①$$

$$S_{28} = \frac{28}{2}(2a + 27d) = 9$$

$$14(2a + 27d) = 9$$

$$a + \frac{27}{2}d = \frac{9}{28} \quad - ②$$

eq ① - eq ① $\Rightarrow$

$$9.5d = \frac{-703}{9 \times 28}$$

$$d = -\frac{37}{126}$$

$$\Rightarrow a = \frac{28}{9} + \frac{37 \times 4}{126}$$

$$a = \frac{30}{7}$$

$$\therefore S = S_{37} = \frac{37}{2}(2a + 36d)$$

$$S = \frac{37}{2}\left[\frac{60}{7} + \frac{(36)(-37)}{126}\right]$$

$$S = \frac{37}{2}\left[\frac{-252}{126}\right]$$

$$S = -37$$

$$\therefore |S| = 37$$

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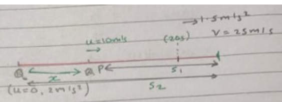
**20**

Q20)




$s_1 + x = s_2$

for $s_1$ we have two parts

① $s_x = \cancel{10(20) + \frac{1}{2}} = 10(20) = 200m$

$s_y \Rightarrow v^2 = u^2 + 2as_y$

$625 = 100 + 2(1.5)s_y$

$\Rightarrow \frac{525}{3} = 175m = s_y$

$\therefore s_1 = s_x + s_y$

$= 375m$

for $s_2$

$u = 0 \quad a = 2m/s^2 \quad t = 20 + 10$ $\quad$ $(25 = 10 + 1.5t$, $t = 10s)$

$= 30s$

$s_2 = 0 + \frac{1}{2} \times 2 \times 900$

$= 900m$ $\quad$ - solution by Arnav Dave

$x = 900 - 375$

$= \boxed{525m}$

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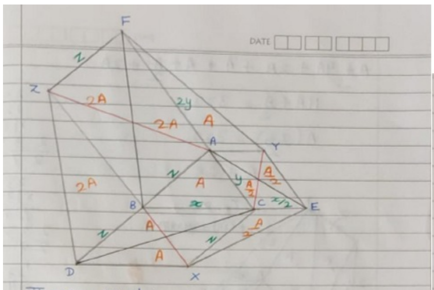
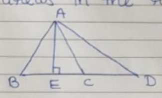
**21**




Theory used:
Two triangle with common vertex have areas in the ratio of their bases.




Given : $\frac{BC}{CD} = \frac{2}{1}$.

Area of triangle ABC : $= \frac{1}{2} \times BC \times AE$.

Area of triangle ACD : $= \frac{1}{2} \times CD \times AE$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ACD} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times CD \times AE}$$

Now, in the given figure,
Ratios of bases are given in GREEN INK.
Let ~~ratios~~ area of triangle ABC be A

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ACE} = \frac{BC}{CE} = \frac{x}{\frac{x}{2}} = \frac{2}{1}$$

$$\frac{A}{\text{Area of } \triangle ACE} = \frac{2}{1}$$

Area of $\triangle ACE = \frac{A}{2}$

As $\square ACEY$ is a parallelogram,
Area of $\triangle ACE$ = Area of $\triangle AYE$

$\Rightarrow$ Area of $\triangle AYE = \frac{A}{2}$

$\Rightarrow$ Area of $\square ACEY = \frac{A}{2} + \frac{A}{2} = A$.

Now, Area of $\triangle ACY$ = Area of $\triangle CYE = \frac{A}{2}$

$$\frac{\text{Area of } \triangle ACY}{\text{Area of } \triangle FAY} = \frac{AC}{AF} = \frac{y}{2y} = \frac{1}{2}$$

$$\frac{\frac{A}{2}}{\text{Area of } \triangle FAY} = \frac{1}{2}$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle FAB} = \frac{AC}{AF} = \frac{y}{2y} = \frac{1}{2}$$

$$\frac{A}{\text{Area of } \triangle FAB} = \frac{1}{2}$$

Area of $\triangle FAB = 2A$

As $\square ABZF$ is a parallelogram,
Area of $\triangle FAB$ = Area of $\triangle ZBF = 2A$

Area of $\square ABZF = 4A$.

Area of $\triangle ABZ$ = Area of $\triangle AFZ = \frac{4A}{2} = 2A$.

$$\frac{\text{Area of } \triangle ABZ}{\text{Area of } \triangle DBZ} = \frac{z}{z} = \frac{1}{1}$$

$$\frac{2A}{\text{Area of } \triangle DBZ} = \frac{1}{1}$$

Area of $\triangle DBZ = 2A$.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle CBD} = \frac{z}{z} = \frac{1}{1}$$

$$\frac{A}{\text{Area of } \triangle CBD} = \frac{1}{1}$$

Area of $\triangle CBD = A$

Now, as $\square DBCX$ is a parallelogram,

Area of $\triangle DBC$ = Area of $\triangle DCX = A$

$\Rightarrow$ Area of $\square DBCX = 2A$.

$\Rightarrow$ Area of $\triangle BCX$ = Area of $\triangle BDX = \frac{2A}{2} = A$

$$\frac{\text{Area of } \triangle BCX}{\text{Area of } \triangle CXE} = \frac{x}{\frac{x}{2}} = \frac{2}{1}$$

$$\frac{A}{\text{Area of } \triangle CXE} = \frac{2}{1}$$

Area of $\triangle CXE = \frac{A}{2}$

Now, area of hexagon DXEYFZ ;

$[\square DBCX] + [\square ACEY] + [\square ABZF] + [\triangle ABC] + [\triangle CXE] + [\triangle FAY] + [\triangle DBZ]$

$$2A + A + 4A + A + \frac{A}{2} + A + 2A = 322$$

$$\frac{23A}{2} = 322$$

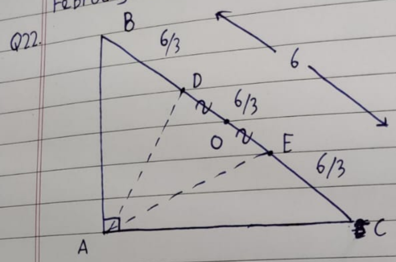
$$A = 28$$

$\Rightarrow$ Area of $\triangle ABC = 28$.

**22**

February 2021 11th Entrance Test

Q22.




$BC = 6$

$\because BD = DE = CE$, $BD = \frac{6}{3} = DE = CE$

We know that the circumcenter of a right triangle lies on the midpoint of the hypotenuse and has radius equal to $OB = OC = OA$

$\because O$ is midpoint of $BC$, it is also midpoint of $DE$

$\Rightarrow OD = \frac{6}{3} \times \frac{1}{2} = 1$ and $OB = OC = OA = BD + DO$

$= \text{radius of circumcircle}$

$= \frac{6}{3} + 1 = 3$ -- (1)

Using Appolonius Thm in $\Delta ADE$, ($\because O$ is midpoint of $DE$ so $OA$ becomes median)

$\therefore AD^2 + AE^2 = 2AO^2 + 2OE^2$

$= 2AO^2 + 2OD^2$

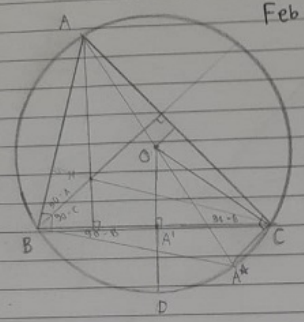
$= 2(3)^2 + 2(1)^2$ -- from (1)

$= 18 + 2$

$= \boxed{20}$

**23**

Feb-2021 Entrance Test
Q 23




Consider $\Delta ABC$.
let $\overrightarrow{AO} \cap \odot ABC = \{A^*\}$
As $AA^*$ is a diameter, $\angle ACA^* = \angle ABA^* = 90°$
$\angle HBC = 90 - C$ . $\angle ABH = 90 - A$ $\angle HCB = 90 - B$
$\angle ABA^* = \angle ABH + \angle HBC + \angle CBA^*$
$90 = 90 - C + 90 - A + \angle CBA^*$
$90 = 180 - (A+C) + \angle CBA^*$
$A + C - 90 = \angle CBA^* \Rightarrow 180 - B - 90 = \angle CBA^*$
$\therefore \angle CBA^* = 90 - B$
$\therefore$ as $\angle CBA = \angle BCH$, $BH \parallel A^*C$
As $BH \perp AC$ and $A^*C \perp AC$ $HC \parallel BA^*$
$\therefore \square HBA^*C$ is a parallelogram
$\therefore$ if $A'$ is midpt of $BC$ $A'$ is also midpt of $HA^*$
$\therefore$ By midpt thrm to $\Delta AHA^*$, $OA' = \frac{1}{2} AH$ & $OA' \parallel AH$
Now consider $\Delta OA'C$
Given $AH = R \Rightarrow OA' = \frac{1}{2} AH = \frac{R}{2}$
$\angle OA'C = 90°$
$\therefore \Delta OA'C$ is 60-90-30 $\Delta$
$OA' = \frac{1}{2} OC \Rightarrow \angle OCA' = 30° \Rightarrow \angle A'OC = 60°$
~~Hence~~ As $\Delta BOA' \cong \Delta COA'$ By S-S-S ($OC = OB$, $BA' = A'C$, $OA'$ co
$\angle BOA' = \angle A'OC$ ~~at~~ $\Rightarrow \angle BOC = 2 \angle A'OC$
$\therefore$ chord $BC$ of $\odot ABC$ subtends $\angle \frac{BOC}{2}$ at pt $A$
classmate

$\therefore \angle BAC = 2\angle A'OC / 2$
$= \angle A'OC$
$= 60°$

## 24

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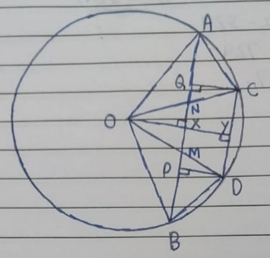


Construction: Drop ~~⊥~~ perpendicolor from C & D ~~resp.~~ on AB at Q & P resp.

: Drop perpendicular from O on CD at Y

~~$OY \cap AB = \{X\}$~~

$OY \cap AB = \{X\}$

$\triangle OBM \cong \triangle OAN$ (SAA Test)

$\therefore \angle BAD = \angle COD = \angle AOC$ —— (1)

As equal angles at centre intercept equal arcs & therefore equal chords we can say that,

$AC = CD = BD = 18$ —— (2)

$QP = CD = 18$ —— (3)

$AQ = PB$ —— (4)

$\therefore AQ = PB = \dfrac{AB - PQ}{2} = 14$

By Pythagoras Theorem

$QC = PD = XY = \sqrt{128} = 8\sqrt{2}$ —— (5)

By Pythagoras theorem

$OX = \sqrt{R^2 - 529}$ $\quad (\because OB = R \ \& \ BX = AB \times 1/2 = 23)$

$OY = \sqrt{R^2 - 81}$ $\quad (\because OD = R \ \& \ DY = CD \times 1/2 = 9)$

$OY - OX = YX = 8\sqrt{2}$

$\sqrt{R^2 - 81} - \sqrt{R^2 - 529} = 8\sqrt{2}$

Let $x = R^2$

$\sqrt{x - 81} = 8\sqrt{2} + \sqrt{x - 529}$

~~Solving we get, $x =$~~

$\cancel{x} - 81 = 128 + \cancel{x} - 529 + 16\sqrt{2} \cdot \sqrt{x - 529}$

$320 = 16\sqrt{2} \cdot \sqrt{x - 529}$

---

## 25

Given $\triangle ABC$ isosceles $\Rightarrow AB = AC = 18$

$MN \| BD \Rightarrow m\angle BEC = m\angle ECN$

By tangent secant theorem $m\angle ECN = m\angle ABC = m\angle ACB$.

$\Rightarrow m\angle ACB = m\angle ADB$, inscribed angles.

$\Rightarrow \triangle ABC \sim \triangle AED \sim \triangle BEC$ and are isosceles

$\Rightarrow \frac{AB}{BC} = \frac{AG}{ED} = \frac{BE}{EC}$

$\Rightarrow BE = 12$ and $EC = 8$

$\Rightarrow AE = AC - EC = 12 - 8 = 4.$

**26**

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Q26) Given: $x = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ & $y = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

Rationalising the denominator,

$$x = \frac{10+2\sqrt{21}}{4} \quad \& \quad y = \frac{10-2\sqrt{21}}{4}$$

$$\therefore x+y = \frac{20}{4} = 5 \quad \text{—(1)}$$

Notice that $y = \frac{1}{x}$ OR $x = \frac{1}{y}$

Concept Used: $\left(k+\frac{1}{k}\right)^2 = k^2+\frac{1}{k^2}+2$, ie. $\left(k+\frac{1}{k}\right)^2-2 = k^2+\frac{1}{k^2}$

Say $E = \sqrt{2(x^4+y^4+(x+y)^4)}$

$$\therefore E = \sqrt{2\left[x^4+\frac{1}{x^4}+(x+y)^4\right]} = \sqrt{2\left[(x^2)^2+\frac{1}{(x^2)^2}+(x+y)^4\right]}$$

$$= \sqrt{2\left[\left(x^2+\frac{1}{x^2}\right)^2-2+(x+y)^4\right]} = \sqrt{2\left[\left[\left(x+\frac{1}{x}\right)^2-2\right]^2-2+(x+y)^4\right]}$$

~~Say~~ Now, let $x+y = x+\frac{1}{x} = 'm'$ (from (1), $m=5$)

$$\therefore E = \sqrt{2\left\{(m^2-2)^2-2+m^4\right\}} = \sqrt{2(m^4-4m^2+m^4-2+4)}$$

$$= \sqrt{2(2m^4-4m^2+2)}$$

$$= 2\sqrt{m^4-2m^2+1} = 2\sqrt{(m^2-1)^2}$$

$$= 2|(m^2-1)| = 2(1-m^2) = 2(1-25) = -48$$

OR

$$2(m^2-1) = 2(25-1) = 48$$

$\therefore E = 48$ OR $-48$

$\Rightarrow \boxed{E = \pm 48}$

## 27

$$\frac{1}{x}+\frac{1}{y+z}=\frac{1}{2},\quad \frac{1}{y}+\frac{1}{z+x}=\frac{1}{3},\quad \frac{1}{z}+\frac{1}{x+y}=\frac{1}{4}$$

$$\Rightarrow \frac{x+y+x}{xy+xz}=\frac{1}{2},\quad \frac{x+y+z}{yz+yx}=\frac{1}{3},\quad \frac{x+y+z}{3x+zy}=\frac{1}{4}$$

$$\Rightarrow xy+xz=2(x+y+z)\cdots(1)$$

$$\Rightarrow yz+yx=3(x+y+z)\cdots(2)$$

$$\Rightarrow zx+zy=4(x+y+z)\cdots(3)$$

Adding (1),(2), (3) we get

$$xy+yz+zx=\frac{9}{2}(x+y+z)$$

(2) - (1) $\Rightarrow yz+yx-xy-xz=x+y+z$

$$\Rightarrow \quad yz-xz=x+y+z\cdots(4)$$

(3) + (4) $\Rightarrow$ $2yz=5(x+y+z)=5\left[\frac{2xy+2yz+2xz}{9}\right]$

$$\Rightarrow xy+yz+zx=\frac{9}{5}yz=\frac{9}{2}(x+y+z)$$

$$\Rightarrow x+y+z=\frac{2}{5}yz$$

(2) $\Rightarrow yz+yx=3\left(\frac{2}{5},yz\right)=\frac{6}{5}yz$

$$\Rightarrow yx=\frac{yx}{5}\Rightarrow x=\frac{z}{5}\Rightarrow z=5y$$

(3) $\Rightarrow zx+zy=4\left(\frac{2}{5}yz\right)=\frac{8}{5}yz$

$$\Rightarrow yx=\frac{3}{5}yz$$

$$\Rightarrow x=\frac{3}{5}y\Rightarrow y=\frac{5}{3}x$$

Putting these values in first eqnation

$$\frac{1}{x}+\frac{1}{\frac{5}{3}x+5x}=\frac{1}{2}\Rightarrow \frac{1}{x}+\frac{3}{20x}=\frac{1}{2}$$

$$\Rightarrow \frac{23}{20x}=\frac{1}{2}\Rightarrow x=\frac{23}{10};y=\frac{23}{10}\times\frac{5}{3}=\frac{23}{6}$$

and $z=5\times\frac{23}{10}=\frac{23}{2}\Rightarrow$ Ans $=18$

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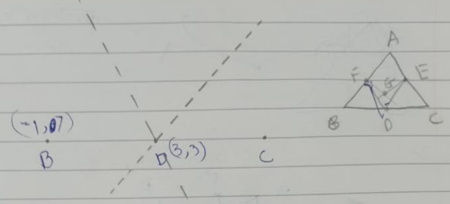
## 28

Entrance exam February 2021-23

Q28




We know AB || ED & AC || FD — (1)

D is mid point of BC.

Let coordinates of C be $(x_c, y_c)$

$\frac{x_c - 1}{2} = 3$ $\quad$ $\frac{y_c + 7}{2} = 3$

$x_c = 7$ $\quad$ $y_c = -1$.

$C \equiv (7, -1)$ — (2)

slope of AC = slope of FD (By (1))

eqn of AC $\rightarrow$ $y = mx + c$

putting C $\quad -1 = \frac{1}{2} \times 7 + c$

$c = -4.5$

eqn of AC

$y = \frac{1}{2}x - 4.5$ — (3)

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## 29

Sum of first 9 terms $= 28$
sum of first 28 terms $= 9$
$\therefore$ sum of 17 terms, $t_{10}$ to $t_{28}$, is $9 - 28 = -17$
$\Rightarrow$ Average (middle term ) $= \frac{-17}{17} = -1 \Rightarrow$ Term No $19 = -1$
which in average of all 37 terms.
$S =$ Sum of 37 terms $= -1 \times 37 = -37$
$|S| = 37$

**30**

February 7, 2021

Mathematics

Q30

Given $P(x) = x^4 + 4x^3 + 12x^2 + 16x + K$

$P(x)$ is a perfect square

Let $P(x) = (x^2 + mx + n)^2$

Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(x^2 + mx + n)^2 = x^4 + m^2x^2 + n^2 + 2 \cdot x^2 \cdot mx + 2 \cdot mx \cdot n + 2 \cdot x^2 \cdot n$

$P(x) \quad = x^4 + 2mx^3 + (m^2 + 2n)x^2 + 2mnx + n^2$

$x^4 + 4x^3 + 12x^2 + 16x + K$

$= x^4 + 2mx^3 + (m^2+2n)x^2 + 2mnx + n^2$

Comparing coefficient of $x$ on both sides, we get

$2m = 4 \longrightarrow m = 2$

$m^2 + 2n = 12$

$m = 2 \quad \therefore m^2 = 4$

$2n = 12 - m^2 = 8 \qquad n = 4$

but $K = n^2$.

$\therefore$ The only value of $K$ is $n^2$

$K = 16$

$\therefore$ Sum of all possible values of $K$ is $16$

Mrunmayi Sawant