**1**

1. When oil is treated with hydrogen and passed over finely divided Nickel or Palladium at 200°C, the hydrogen molecules are added to unsaturated carbon – Carbon bonds. Hence, fats are obtained.

Nickel belongs to $4^{th}$ period.
Palladium belongs to $5^{th}$ period.

Atomic number of Nickel = 28.

**2**

Q2. Elements belonging to $13^{th}$ group are:

B (Boron)
Al (Aluminium)
Ga (Gallium)
In (Indium)
Tl (Thallium)
Nh (Nihonium)
Out of these elements only Ga (Gallium) exists in liquid state at room temperature

Atomic ~~Mas~~ Number of Gallium = $\boxed{31}$

Ans 31

– Solution by Parth Raghashetty

**3**

Chemistry
3) Lanthanides → Group '3' and period '6'
$\Rightarrow X = 3; Y = 6$
Actinides → Group '3' and period '7'
$\Rightarrow X = 3; Y = 7$

$X + Y + Z = 3 + 6 + 7 = 16$

– Solution by SaiKrish K

**4**

i) $KNO_{3(s)} + H_2O \rightarrow KNO_{3(aq)}$

Endothermic Reaction as it is creating aqeous solution which results in breaking of $H_2O \rightarrow OH^-$, $H^+$

As Breaking of Bonds = Endothermic

∴ It is an endothermic reaction

ii) $C_6H_{12}O_{6(aq)} + 6O_{2(g)} \rightarrow 6CO_{2(g)} + 6H_2O_{(l)}$

Combustion Reaction = Exothermic

iii) $NH_4Cl_{(s)} + H_2O \rightarrow NH_4Cl_{(aq)}$

Dissolution = Endothermic

iv) $S + O_{2(g)} \rightarrow SO_{2(g)}$

Making of Bond = Exothermic reaction

v) $CaCO_{3(s)} \rightarrow CaO_{(s)} + CO_{2(g)}$

Endothermic Reaction = Decomposition

vi) $C_{12}H_{22}O_{11} \rightarrow 12C + 11H_2O_{(l)}$

Endothermic

vii) Addition of NaCl to water ⇒ Endothermic

viii) Sodium metal in water is an explosive reaction which liberates a huge amt. of heat

∴ Exothermic    ∴ Ans = $\boxed{5}$ Endothermic reactions

**5**

Q.5) Solution:

i) $\overset{-4}{C}H_{4(g)} + \overset{0}{O}_{2(g)} \longrightarrow \overset{+4}{C}\overset{-2}{O}_{2(g)} + H_2\overset{-2}{O}_{(l)}$

⇒ C is oxidised and O reduced

⇒ It is a redox reaction.

ii) $\overset{+1}{H}_2O + \overset{+4}{C}\overset{-2}{O}_{2(g)} \longrightarrow \overset{+1}{H}_2\overset{+4}{C}\overset{-2}{O}_{3(aq)}$

⇒ It is not a redox reaction.

iii) $\overset{+1}{Ag}\overset{+5}{N}\overset{-2}{O}_3 + \overset{+1}{Na}\overset{-1}{Cl} \longrightarrow \overset{+1}{Na}\overset{+5}{N}\overset{-2}{O}_3 + \overset{+1}{Ag}\overset{-1}{Cl}$

⇒ It is not a redox reaction.

iv) $\overset{+2}{Ba}\overset{+6}{S}O_4 + 4\overset{0}{C} \longrightarrow \overset{+2}{Ba}\overset{-2}{S} + 4\overset{+2}{C}\overset{-2}{O}$

⇒ S is reduced and C oxidised

⇒ It is a redox reaction

v) $C_2H_5O\overset{+1}{H} + \overset{0}{Na} \longrightarrow C_2H_5O\overset{+1}{Na} + \overset{0}{H}_2\uparrow$

⇒ It is a redox reaction.

vi) $2H\overset{-1}{Cl} + Ca(OH)_2 \longrightarrow Ca\overset{-1}{Cl}_2 + 2H_2O$

⇒ It is not a redox reaction.

vii) $4\overset{0}{Al} + 3\overset{0}{O}_2 \longrightarrow 2\overset{+3}{Al}_2\overset{-2}{O}_3$

⇒ It is a redox reaction.

viii) $\overset{0}{Zn} + 2NaO\overset{+1}{H} \longrightarrow Na_2\overset{+2}{Zn}O_2 + \overset{0}{H}_2\uparrow$

⇒ It is a redox reaction.

ix) $Ba\overset{-2}{S} + Zn\overset{+6}{S}O_4 \longrightarrow Ba\overset{+6}{S}O_4 + Zn\overset{-2}{S}$

⇒ It is not a redox reaction.

⇒ Reactions 2,3,6,9 are non redox

⇒ Ans: $\boxed{4}$

—— Solution by Advay Sangrulkar

**6**

Q6 Carbonate of 'A' → $ACO_3$ | Let $(MM)_A = a$

Nitride of 'B' → $B_3N_2$ | $(MM)_B = b$

Molar mass of $ACO_3 = (MM)_A + 12 + 48$

$= a + 60$

Molar mass of $B_3N_2 = 3(MM)_b + 2(14)$

$= 3b + 28$

Given → $3b + 28 = a + 60$ → $3b - a = 32$

| Elements belonging to Group 2 : | Be | Mg | Ca | Sr | Ba | Ra |
|---|---|---|---|---|---|---|
| Their atomic mass : | 9 | 24 | 40 | 87 | 137 | 226 |

We can see that $b = 24$, $a = 40$ satisfy the equation.

∴ A = Ca

B = Mg

$a - b = 40 - 24 = \underline{\underline{16}}$

— Mohit Soundankar

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**7**

3 Jan 2021 Paper

Chemistry Q. 7.

7. $3C_2H_5OH + PCl_3 \longrightarrow 3C_2H_5Cl + H_3PO_3$

Molar mass of $H_3PO_3$

$= 3\times1 + 31 + 3\times16$

$= 3 + 31 + 48 = \boxed{82}$

Solution by Arya Gujar

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**8**

Q8) 1) Zn

$Zn + \text{dil } HNO_3 \rightarrow Zn(NO_3)_2 + \underline{N_2O} + H_2O$

$\therefore N_2O$ is formed

$Zn + \text{conc } HNO_3 \rightarrow Zn(NO_3)_2 + \underline{NO_2} + H_2O$

2) Cu

$Cu + \text{dil } HNO_3 \rightarrow Cu(NO_3)_2 + \underline{NO_2} + H_2O$

$Cu + \text{conc } HNO_3 \rightarrow Cu(NO_3)_2 + \underline{NO_2} + H_2O$

3) Ag

$Ag + \text{dil } HNO_3 \rightarrow AgNO_3 + \underline{NO_2} + H_2O$

$Ag + \text{conc } HNO_3 \rightarrow AgNO_3 + \underline{NO_2} + H_2O$

4) Hg

$Hg + \text{dil } HNO_3 \rightarrow Hg(NO_3)_2 + \underline{NO_2} + H_2O$

$Hg + \text{conc } HNO_3 \rightarrow Hg(NO_3)_2 + \underline{NO_2} + H_2O$

5) Ca

$Ca + \text{dil } HNO_3 \rightarrow Ca(NO_3)_2 + H_2O + \underline{N_2O}$

$Ca + \text{conc } HNO_3 \rightarrow Ca(NO_3)_2 + H_2O + \underline{NO_2}$

6) $Au + \text{any conc of } HNO_3 \rightarrow$ No Reaction

7) $Pt + \text{any conc of } HNO_3 \rightarrow$ No Reaction

$\therefore$ 5 Elements produce atleast one of the oxides : $NO$, $NO_2$ & $N_2O$ when reacted with $HNO_3$

$\therefore \boxed{ANS = 5}$

- Solution By Bhargavi Deo.

**9**

Q9.

After electrolytic reduction,
NaCl
becomes $Na^+$ & $Cl^-$ ions

At cathode,
1 mole Na is deposited
So $Na^+ + e^- \rightarrow Na$
Hence 1 mole of electrons required
So $x = 1$

At anode,
1 mole of $Cl_2$ gas is deposited
$2Cl^- + 2e^- \rightarrow Cl_2$
Hence, 2 mole of electrons required
So $y = 2$

$\therefore x + y$
$= 1 + 2$
$= \boxed{3}$

- Solution by Suhan Chelekar
$X^{th}$ Oly 2022/2023

10 Q. 10]

Solution:

Sodium bicarbonate and sodium carbonate react with vinegar to produce $CO_2$:-

$$Na_2CO_3 + 2CH_3COOH \longrightarrow 2CH_3COONa + H_2O + CO_2$$

$$NaHCO_3 + CH_3COOH \longrightarrow CH_3COONa + H_2O + CO_2$$

Let number of mole of $Na_2CO_3$ and $NaHCO_3$ be $x$

$\therefore$ In the first reaction, $x$ mole of $CO_2$, $x$ mole of $H_2O$, and $2x$ mole of $CH_3COONa$ are produced

$\Rightarrow$ Molar percentage of $CO_2$ $= \frac{x}{4x} \times 100 = 25\%$

$\therefore$ In the second reaction, $x$ mole of $CO_2$, $x$ mole of $H_2O$, and $x$ mole of $CH_3COOH$ are produced

$\Rightarrow$ Molar percentage of $CO_2 = \frac{x}{3x} \times 100 = 33.33\%$

$\Rightarrow$ % of $CO_2$ is greater in $NaHCO_3$

$\Rightarrow A = NaHCO_3$

$\therefore M =$ molecular mass of $NaHCO_3 = (23+1+12+3\times16)$

$\therefore M = 84$ g/mol

$\Rightarrow \frac{M}{2} = \frac{84}{2} = \boxed{42}$

— Solution by
Aryansingh Sonaye
X Olympiad 2022-23

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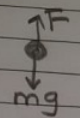
11 Solution by Soham Pate

11. Let $m = 425$ kg.

Let $F$ be the buoyant force.

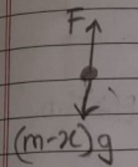
Initially,




acceleration $= 0$ m/s$^2$

$\Rightarrow F = mg$

Now, suppose, $x$ mass is removed.




acceleration $= 0.18$ m/s$^2$ upwards

$\Rightarrow F - (m-x)g = (m-x)a$
$= (m-x)(0.18)$

$\Rightarrow 10x = -0.18x + 425 \times 0.18$

$\Rightarrow x \approx 7.5$ kg

$\therefore 2x \approx 15$

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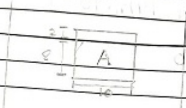
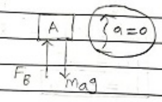
**12** Q12 ..., 2021 paper

I




FBD of A:




Here,

$F_B = m_a g$

$F_B =$ Vol. displaced $\times \rho_{liq} \times g$

Vol. disp. $= 800\ cm^3 = 8 \times 10^{-4}\ m^3$

$\rho_{liq} = \rho_{water} = 1000\ kg/m^3$

$m_a g = Vol_{(a)} \times \rho_a \times g$

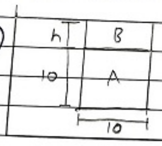
$Vol_{(a)} = 1000\ cm^3 = 10^{-3}\ m^3$

$\Rightarrow 8 \times 10^{-4} \times 10^3 \cancel{g} = 10^{-3} \times \rho_a \times \cancel{g}$

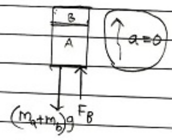
$\Rightarrow \rho_a = \dfrac{8 \times 10^{-1}}{10^{-3}} = 800\ kg/m^3$

$\Rightarrow \boxed{\rho_a = 800\ kg/m^3}$

II




FBD:




Here,

$F_B = (m_a + m_b) g$

$F_B =$ Vol. disp $\times \rho_{water} \times g$

Vol. disp $= (10+h) \times 100\ cm^3 = (10+h) \times 10^{-4}\ m^3$

$(m_a + m_b) g = (Vol_{(a)} \times \rho_a + Vol_{(b)} \times \rho_b) g$

$Vol_{(b)} = 100h\ cm^3 = h \times 10^{-4}\ m^3$

$\rho_b = 1400\ kg/m^3$

$\Rightarrow (10+h) \times 10^{-4} \times 1000 \times \cancel{g} = ((10^{-3})(800) + (h \times 10^{-4})(1400)) \cancel{g}$

$\Rightarrow (10+h) \times 10^{-1} = [8 \times 10^{-1} + 14h \times 10^{-2}]$

$\Rightarrow \dfrac{(10+h)10}{100} = \dfrac{80}{100} + \dfrac{14h}{100}$

$\Rightarrow 100 + 10h = 80 + 14h$

$\Rightarrow 4h = 20$

$\Rightarrow \boxed{h = 5cm}$

Hence the height of new block B is 5 cm.

– Solution by Parth Raghashetty

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**13** Physics

13) 

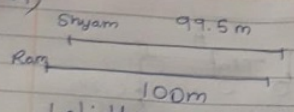
| Heat Gained | Heat Released |
| --- | --- |
| 24g Ice at 0°C → 24g water at 0°C | ∴ 4320 cal should be released |
| $\Rightarrow Q = mL_f = 24 \times 80$ | $\Rightarrow Q = mL_f$ |
| $= 1920 cal$ | $\Rightarrow 4320 = m \cdot 540$ |
| 24g water at 0°C → 24g water at 100°C | $\Rightarrow m = 8g$ |
| $\Rightarrow Q = mc\Delta T = 24 \times 1 \times 100$ | ∴ 8g water at 100°C formed |
| $= 2400 cal$ | and rest 2g steam at 100°C |
| Total - 4320 cal | |

∴ Mass of water in Container $= 8 + 24 = 32g$

– Solution by Sai Krish K

**14**

14)




Let the speed of Shyam $= V_S$

the speed of Ram $= V_R$

According to the 1st condition

$$\frac{1}{5} = \frac{99.5}{V_S} - \frac{100}{V_R} \quad \text{---} (I)$$

$\frac{99.5}{V_S}$ ↑ time of Shyam; $\frac{100}{V_R}$ ↑ time of Ram

According ~~of~~ to the 2nd condition

$$\frac{1}{5} = \frac{100}{V_R} - \frac{95.5}{V_S} \quad \text{---} (II)$$

Solving I & II

$$\frac{99.5}{V_S} - \frac{100}{V_R} = \frac{100}{V_R} - \frac{95.5}{V_S}$$

$$\therefore \frac{195}{V_S} = \frac{200}{V_R} \Rightarrow \boxed{V_S = \frac{39}{40} V_R}$$

$$\therefore \frac{1}{5} = \frac{40 \times 99.5}{39 V_R} - \frac{100}{V_R}$$

$$\therefore \frac{1}{5} = \frac{3980 - 3900}{39 V_R}$$

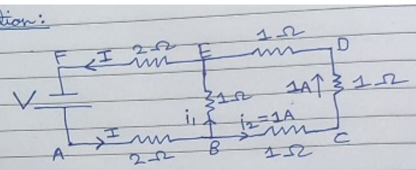
$$\therefore \boxed{V_R = \frac{400}{39}}$$

$\therefore$ Time of Ram for 100 meter race $= \frac{100 \times 39}{400} = \boxed{\frac{39}{4}}$

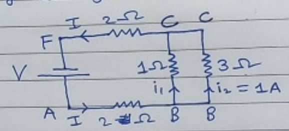
$$\therefore \frac{39}{4} \times 4 = \boxed{39}$$

**15**

Q.15) Solution:




Redrawing the given circuit to get the $R_{eff}$ →




$\Rightarrow R_{eff} = 2 + (1//3) + 2$

$\Rightarrow R_{eff} = 4.75\ \Omega$

$\Rightarrow V = I \times R_{eff}$

$\Rightarrow I = \frac{V}{4.75}$ — ①

Now, by Junction rule at point B →

$I = i_1 + i_2$ — ②

Also, by calculating $V_{CB}$ in 2 ways,

$3 \times i_2 = 1 \times i_1$

$\Rightarrow i_1 = 3 i_2 = 3(1) = 3A$ — given that $i_2 = 1A$

$\Rightarrow$ From ① → $I = i_1 + i_2 = \frac{V}{4.75}$

$\Rightarrow 4 = \frac{V}{4.75}$

$\Rightarrow V = 4.75 \times 4 = 19V$

$\Rightarrow 4V = 19 \times 4 = \boxed{76V}$

Ans: $\boxed{76}$.

— Solution by
Advay Sangrulkar

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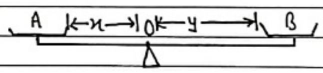
**16**

Paper date : 3 January 2021
Physics

Q16]




A defective balance has different distances of the pans from the center mass balance. (i.e. point O)
The ratio of distance of the pans from the point O is the ratio of the weight kept in it.
Let the actual weight of the wheat be M.

Case 1: Wheat bag kept at A.
16 Kg was weighed at pan B.

$\frac{x}{y} = \frac{M}{16}$ — ①

Case 2: Wheat bag kept at B.
9 Kg was weighed at A.

$\frac{x}{y} = \frac{9}{M}$ — ②

By ① and ② → $\frac{M}{16} = \frac{9}{M} \Rightarrow M^2 = 144 \Rightarrow M = 12$ Kg

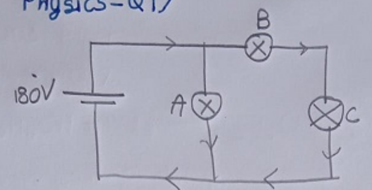
Extra money paid $= (12.5)30 - 12(30) = \underline{\underline{15}}$ rupees

— Mohit Soundankar

**17**

3 Jan 2021

Physics – Q17




$P = \frac{V^2}{R}$  $\therefore R_A = \frac{V^2}{P} = \frac{240^2}{24} = 2400\,\Omega$

$R_B = \frac{240^2}{60} = 960\,\Omega$

$R_C = \frac{240^2}{40} = 1440\,\Omega$

$R_{eff} = R_A \parallel (R_B + R_C)$

$= 2400 \parallel 2400 = 1200\,\Omega$

$\therefore I = \frac{V}{R} = \frac{180}{1200} = 0.15\,\Omega$

$1/R \propto I$ [R is equal i.e. 2400]  $\therefore$ Current through $A = \frac{0.15}{2} = \frac{3}{40}\,\Omega$

similiarly, current through B & C $= \frac{3}{40}\,\Omega$

For C: $P = I^2 R$

$= \frac{3}{40} \times \frac{3}{40} \times 1440$

$= 8.1\,W$

Solution by Arya Gujar

$\therefore 10P = \boxed{81W}$

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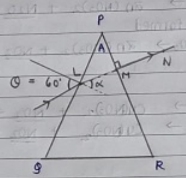
**18**

Q18) (Refractive Index) glass $= n$

$A = 30^\circ$




The Emergent ray makes an angle 0° with the Normal [$\overrightarrow{MN}$]

$\therefore$ The Refracted ray [$\overrightarrow{LM}$] also makes an angle 0° with the Normal

$\therefore \angle PML = 90$

$A + \angle PML + \angle PLM = 180^\circ$ [Angle sum Property]

$\therefore \angle PLM = 60^\circ$

$\angle PLM + \alpha = 90^\circ \Rightarrow \alpha = 30^\circ$

$\therefore$ incident angle $= \theta = 60^\circ$

Refracted angle $= \alpha = 30^\circ$

Let (Refractive Index) Air $= n' = 1$

By sine Rule

$n' \sin\theta = n \sin\alpha$ [By Snell's law]

Substitute values to get

$n = \frac{1 \times \sin 60}{\sin 30} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

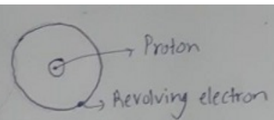
$\therefore n = \sqrt{3} \Rightarrow$ Squaring & Multiplying by 4 we get

$\boxed{Ans = 12}$

– Solution By Bhargavi Deo

**19**

Q19. Hydrogen atom:-




$p^+$ —— $0.1\,nm = 10^{-10}\,m$ —— $e^-$

$q_p = 1.6\times10^{-19}\,C$ $\quad q_e = 1.6\times10^{-19}\,C$

$m_p = 1.6\times10^{-27}\,kg$ $\quad m_e = 9\times10^{-31}\,kg$

$F_e$ = Colomb force $= \frac{kq_1q_2}{r^2}$

$F_g$ = Gravitational force $= \frac{Gm_1m_2}{r^2}$

$$\therefore \frac{F_e}{F_g} = \frac{kq_1q_2}{r^2} \times \frac{r^2}{Gm_pm_e}$$

$$= \frac{\cancel{9}\times10^{9}\times\cancel{1.6}\times10^{-19}\times1.6\times10^{-19}}{6.67\times10^{-11}\times\cancel{1.6}\times10^{-27}\times\cancel{9}\times10^{-31}}$$

$$= \frac{1.6}{6.67}\times10^{40}$$

$$= 0.24\times10^{40}$$

$$= \boxed{2.4\times10^{39}}$$

$\therefore 10^x = 10^{39}$

$\boxed{x = 39}$

- Solution by Sohan Chelekar
$X^{th}$ Oly - 2022/2023

**20**

Q.20]

Solution :

The object is given velocity v m/s at point A (10 m from ground) such that it just reaches maximum height on the path i.e. point C (17 m from ground)

$\therefore$ Let mass of object be m

$\therefore$ Initial Potential Energy (P.E.) of object $= mgh_1 = 10\,mg$

$\therefore$ Final P.E. of object $= mgh_2 = 17mg$

$\Rightarrow$ Work done by v on the object in vertical direction $= 17\,mg - 10\,mg = 7mg$

Now, the same velocity v is imparted to another object at point B.

$\therefore$ Initial P.E. of object $= mgh_3 = 7mg$

Since same v is imparted

$\Rightarrow$ Work done in vertical direction is same i.e. $7mg$

$\Rightarrow$ Change in P.E. of object $= 7mg$

$\therefore$ Final P.E. = Initial P.E. + Change in P.E.

$= 7\,mg + 7mg$

$\Rightarrow$ Final P.E. $= 14\,mg$

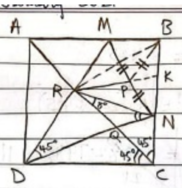
$\Rightarrow$ Height (maximum) at which it has reached

$= \boxed{14}$ m

– Solution by
Aryansingh Sonaye
X Olympiad 2022-23

**21**

Q1.




Join BR, PB.

Let $\overrightarrow{RP} \cap BC = \{K\}$

B is circumcenter of $\Delta MBN$.

$\therefore MP = BP = PN$

$\Delta ARD \cong \Delta ARB$ by SAS criterion.

$\therefore DR = BR$ — ①

$\angle RDN = \angle RCN = 45°$

$\therefore \square DRNC$ is cyclic

$\therefore \angle RND = \angle RCD = 45°$

$\therefore \Delta RDN$ is isosceles.

$\Rightarrow RD = RN$ — ②

① & ② $\Rightarrow$ $BR = RN$ & $PB = PN$

$\therefore \Delta BPR \cong \Delta NPR$ by SSS criterion.

$\therefore \overrightarrow{RK}$ is angle bisector of isosceles $\Delta BRN$.

$\Rightarrow \overrightarrow{RK} \perp \overleftrightarrow{BC}$

$\Rightarrow \overrightarrow{RK} \parallel \overleftrightarrow{DC}$

$\therefore \angle KRC = \angle ACD = 45°$ (Alternate interior angles)

$\therefore \angle PRN = \angle PRC - \angle NRC$

$= 45° - 8°$

$= 37°$

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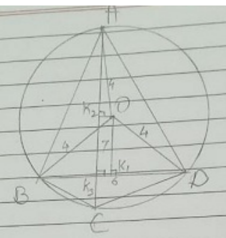
**22**

2021




$BK_1 = DK_1 = 3$

$OK_1 = \sqrt{4^2 - 3^2} = \sqrt{7}$

$OK_2 = K_1K_3$

$AK_2 = CK_2 = \frac{7}{2}$

$OK_2 = \sqrt{16 - \frac{49}{4}}$

$= \sqrt{\frac{64-49}{4}}$

$= \sqrt{\frac{15}{4}}$

$BK_3 = 3 - \sqrt{\frac{15}{4}}$, $DK_3 = 3 + \sqrt{\frac{15}{4}}$

$AK_3 = \frac{7}{2} + \sqrt{7}$

$CK_3 = \frac{7}{2} - \sqrt{7}$

$\frac{AB^2 + BC^2 + CD^2 + DA^2}{2} = CK_3^2 + AK_3^2 + DK_3^2 + K_3B^2$

$= 9 + \frac{15}{4} + 9 + \frac{15}{4} + \frac{49}{4} + \frac{49}{4} + \ldots$

$= 32 + \frac{15}{2} + \frac{49}{2} = 64$

– Shreyash Balshetwar

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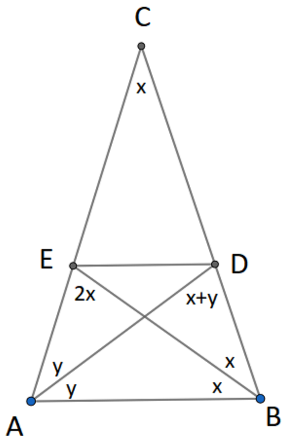
## 23




Let $\overline{BE}$ be angle bisector of $\angle B$.
As $E$ is point on angle bisector of $\angle ABC$
it in equidistant from $\overrightarrow{BA}$ and $BC$
$\Rightarrow$ Area of $\triangle ABE =$ Area of $\triangle DCE$.
Note. $AB = DC$ and $m\angle ABE = M\angle DCE$
Hence it can be proved that $\triangle ABE \cong \triangle DCE$.
Let $m\angle BAC = 2y \Rightarrow m\angle BAD = y$ and $m\angle DAC = y$
$\Rightarrow m\angle ADB = x + y, m\angle ADC = 2x + y$
Congruency gives $m\angle EDC = 2y$ and $m\angle DEC = 2x$
$\Rightarrow m\angle ADE = m\angle ADC - m\angle EDC$

$$= 2x + y - 2y = 2x - y$$

But $m\angle ADB + m\angle ADE + m\angle EDC = 180 \Rightarrow x + y + 2x - y + 2x = 180 \Rightarrow 5x = 180$
$\Rightarrow x = 36. \ln \triangle ABC 2y + 3x = 180$
$\Rightarrow 2y = 72 \quad Ans = 72$

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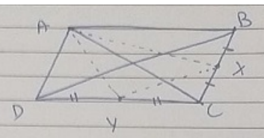
## 24

Q:24




Given: $[ABCD] = 64$

Now; $DY = \frac{1}{2} CD$

$\therefore \frac{[ADY]}{[ADC]} = \frac{1}{2} \Rightarrow [ADY] = 16$ (ratio of base = ratio of area)

Similarly $[AXB] = 16$

Now; $\triangle CBD \sim \triangle CXY$ (by BPT)

Also $\frac{CX}{CB} = \frac{1}{2}$ (X is midpoint)

$\therefore \frac{[BCD]}{[XCY]} = \left(\frac{2}{1}\right)^2 = 4$

but $[BCD] = 32 \Rightarrow [CXY] = 8$

Now $[AXY] + [ADY] + [AXB] + [CXY] = 64$

$\therefore 16 + 16 + 8 + [AXY] = 64$

$\therefore \boxed{[AXY] = 24}$

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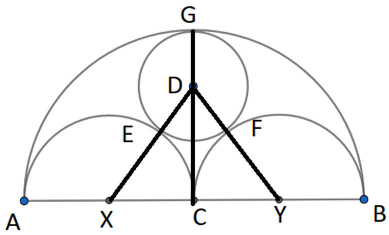
## 25




Let radius of required circle be $r$
Note $CG = 24, XE = 12, XC = 12$
$\Rightarrow CD = 24 - 3, XD = 12 + r$
Aplying Pythagoras to $\triangle DCX$ we get

$$(24 - r)^2 + 12^2 = (12 + r)^2$$
$$\Rightarrow r = 8.$$

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## 26

Q.26

$\therefore \sqrt{x+3} - \sqrt{x - \sqrt{x-2}} = 1$

$\therefore \sqrt{x-\sqrt{x-2}} = \sqrt{x+3} - 1$

$\therefore \cancel{x} - \sqrt{x-2} = \cancel{x} + 3 + 1 - 2\sqrt{x+3}$

$\therefore -\sqrt{x-2} = 4 - 2\sqrt{x+3}$

$\therefore 2\sqrt{x+3} - \sqrt{x-2} = 4$

$\therefore 4(x+3) + (x-2) - 4\sqrt{(x+3)(x-2)} = 4$

$\therefore (4x + x + 12 - 2 - 16)^2 = 16(x+3)(x-2)$

$\therefore (5x-6)^2 = 16(x+3)(x-2)$

$\therefore 25x^2 - 60x + 36 = 16x^2 + 16x - 96$

$\therefore 9x^2 - 76x + 132 = 0$

$\therefore x = \dfrac{76 \pm \sqrt{76^2 - (9)(132)(4)}}{18} \qquad = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{76 \pm 32}{18}$

$= 6 \quad \text{or} \quad 2.44$

$\therefore \sqrt{x-2}$ must be (+ve)

$\therefore x \geqslant 2$, thus as $x \geqslant 2$ it has 2 sol$^n$ 6 & 2.44

$\therefore$ Product of Roots $= \dfrac{132}{9} = K$

$\therefore 3K = \dfrac{132}{9} \cdot 3 = 44$

Ans $= 44$

– Ishaan Anil Karwa

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**27**

Q.27)

$$\begin{array}{r|l} & \frac{a}{3}x^2 + \left(\cancel{b+\frac{a}{3}}\right)/3\, x + \cancel{27}\left(\frac{108+3b-2a}{\cancel{9}27}\right). \\ 3x^2 - x + 1 & ax^4 + bx^3 + 12x^2 - 6x + 1 \end{array}$$

$$ax^4 - \frac{a}{3}x^3 + \frac{a}{3}x^2$$

$$\left(\frac{b}{} + \frac{a}{3}\right)x^3 + \left(12 - \frac{a}{3}\right)x^2 - 6x$$

$$\ominus \quad \left(b + \frac{a}{3}\right)x^3 - \frac{\left(b + \frac{a}{3}\right)}{3}x^2 + \frac{\left(b + \frac{a}{3}\right)}{3}x$$

$$\left(\frac{36-a}{3}\right)x^2 + \left(\frac{3b+a}{9}\right)x^2 - x\left(6 + \frac{3b+a}{9}\right) + 1$$

$$\equiv \left(\frac{108-3a+3b+a}{9}\right)x^2 - \left(\frac{54+3b+a}{9}\right)x + 1$$

$$\ominus \quad \left(\frac{108+3b-2a}{9}\right)x^2 - \left(\frac{108+3b-2a}{27}\right)x + \left(\frac{108+3b-2a}{27}\right)$$

$$\oplus \qquad \ominus$$

$$\left(\frac{108+3b-2a-162-9b-3a}{27}\right)x + \left(1 - \left(\frac{108+3b-2a}{27}\right)\right).$$

$\therefore$ Remainder $= 0$.

$\Rightarrow -6b - 5a - 54 = 0$

$\Rightarrow 6b + 5a = (-54)$

$(-3b + 2a = 81) \times 2$

and $27 - 108 - 3b + 2a = 0$

$\Rightarrow 2a - 3b = 81.$

$$\begin{aligned} 6b + 5a &= (-54) \\ -6b + 4a &= 162 \\ \hline 9a &= 108 \end{aligned}$$

$\Rightarrow \boxed{a = 12}$

$\Rightarrow 6b + 60 = (-54)$

$\Rightarrow 6b = -114 \Rightarrow \boxed{b = -19}$

$\Rightarrow a - (b) = \boxed{31}$ Ans.

Signed by - Punit (F

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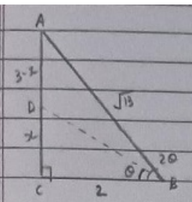
**28**

Q.28.




Given:

$\triangle ABC$ is right angled at C

$\angle ABC = 2\theta$

$2\sec 2\theta = \sqrt{13}$

Solution:

$2\sec 2\theta = \sqrt{13}$

$\therefore \sec 2\theta = \frac{\sqrt{13}}{2} \Rightarrow \cos 2\theta = \frac{2}{\sqrt{13}}$

$\therefore$ In $\triangle ABC$, $BC = 2$, $AB = \sqrt{13} \Rightarrow AC = \sqrt{AB^2 - BC^2} = \sqrt{13-4} = 3$ ... By pythagoras thm

Let BD be angle bisector of $\angle ABC$ and $CD = x$

$\therefore$ Using internal angle bisector theorem,

$$\frac{CD}{AD} = \frac{BC}{AB}$$

$$\Rightarrow \frac{x}{(3-x)} = \frac{2}{\sqrt{13}}$$

$\therefore \sqrt{13}\,x = 6 - 2x$

$\therefore x = \frac{6}{(2+\sqrt{13})}$

$$\tan\theta = \frac{CD}{BC} = \frac{x}{2} = \frac{6}{(2+\sqrt{13})} \times \frac{1}{2} = \frac{3}{(2+\sqrt{13})}$$

$$\therefore (3\tan\theta + 2)^2 = \left[3\left(\frac{3}{2+\sqrt{13}}\right) + 2\right]^2 = \left(\frac{9+4+2\sqrt{13}}{2+\sqrt{13}}\right)^2 = \left[\frac{(\sqrt{13})(2+\sqrt{13})}{(2+\sqrt{13})}\right]^2$$

$$= 13$$

$\therefore$ Answer $= 13$

- Aditya Dhondse

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## 29

Let speed of $A$ be $A/$ day & B be B/day $\Rightarrow$ Total work $= (A+B)\cdot(7.5)$
Half the work is done by A and remaining half done by B in 20 days, days taken by A plus days taken by B is 20

$$\begin{aligned}
&\frac{15A+15B}{4}\left(\frac{1}{A}+\frac{1}{B}\right)=20\\
&(3A+3B)\left(\frac{A+B}{AB}\right)=16\\
&\left(\frac{3A+3B}{B}\right)=16\left(\frac{A}{A+B}\right)=\frac{16}{\left(1+\frac{B}{A}\right)}\\
&\Rightarrow 3+\frac{3}{B/A}=\frac{16}{1+B/A}\\
&Let\frac{B}{A}=t\\
&\Rightarrow 3+\frac{3}{t}=\frac{16}{1+t}\\
&\Rightarrow 3t^2+3t+3+3t-16t=0\\
&\Rightarrow 3t^2-10t+3=0\\
&\Rightarrow 3t^2-9t-t+3=0\\
&\Rightarrow 3t(t-3)-1(t-3)=0\\
&\Rightarrow (3t-1)(t-3)=0\\
&\Rightarrow t=\frac{1}{3}ort=3,\frac{B}{A}=\frac{1}{3}
\end{aligned}$$

As $A$ is more efficient than $B, t=\frac{1}{3}$

$$\begin{aligned}
\text{Total work }&=\frac{(A+B)15}{2}=\frac{B\left(1+\frac{A}{B}\right)}{2}=\frac{B\,(1+3)}{2}\\
&=\frac{B(4)15}{2}
\end{aligned}$$

Days required if B completes alone

$$=\frac{30B}{B}=30.$$

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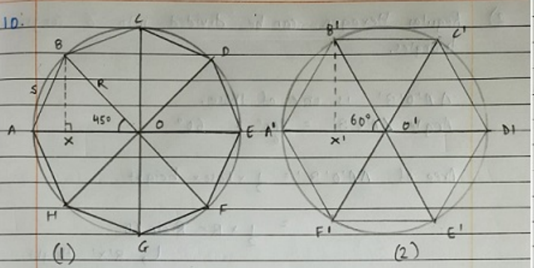
**30**

10.




1) Regular Octagon can be divided into 8 congruent traingles. (As shown in fig 1)
[Congruent since $\underbrace{R, R}_{\text{Radii}}, \underbrace{S}_{\text{side}}$ all equal) → SSS criteria]

$\Delta AOB$ is one of those triangles.

Angle $\angle AOB = \frac{360°}{8}$ ← Total angle around a pt ← 8 divisions for 8 triangles

$= 45°$

Area of $\Delta AOB = \frac{1}{2} \times$ base $\times$ height

Height $BX = R\sin 45$ [Using $\Delta BOX$]

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times R \times R\sin 45 = \frac{R^2}{2} \times \frac{1}{\sqrt{2}} = \frac{R^2}{2\sqrt{2}}$ (1)

2) Regular Hexagon can be divided into 6 congruent triangles.

$\Delta A'O'B'$ is one of those.

Angle $\angle A'O'B' = \frac{360°}{6} = 60°$

Area of $\Delta A'O'B' = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times R \times R\sin 60$

↳ $B'X'$ Height $= R\sin 60$ from $\Delta B'O'X'$

$= \frac{R^2}{2} \times \frac{\sqrt{3}}{2} = \frac{R^2}{4}\sqrt{3}$ (2)

A = Area of Octagon

$= 8 \times$ (1) $= 8 \times$ Area of $\Delta AOB$

$= 8 \times \frac{R^2}{2\sqrt{2}} = R^2(2\sqrt{2})$

B = Area of Hexagon

$= 6 \times$ (2) $= 6 \times \frac{R^2}{4}\sqrt{3} = \frac{3\sqrt{3}}{2} R^2$

Thus $27\left(\frac{A}{B}\right)^2 = 27\left(\frac{\cancel{R^2}\, 2\sqrt{2}}{3\sqrt{3}\,\cancel{R^2}} \times 2\right)^2$

$= 27\left(\frac{4\sqrt{2}}{3\sqrt{3}}\right)^2 = (4\sqrt{2})^2$

$= \underline{\underline{32}}$

- Aksh Shah