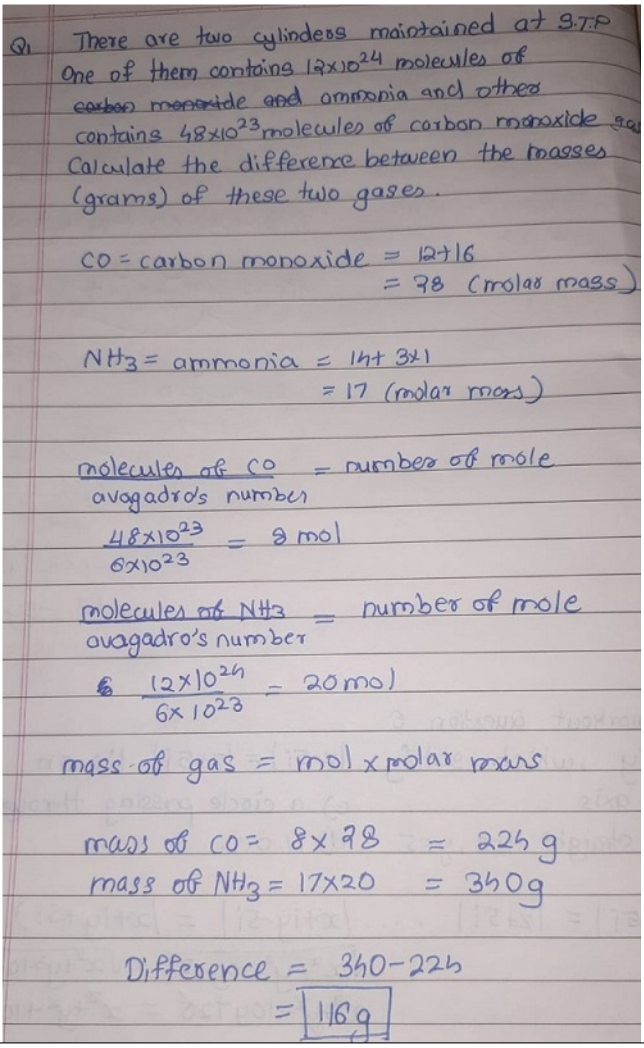
1



Q1 There are two cylinders maintained at S.T.P One of them contains 12×10^{24} molecules of ~~carbon monoxide and~~ ammonia and other contains 48×10^{23} molecules of carbon monoxide ga Calculate the difference between the masses (grams) of these two gases.

CO = carbon monoxide = 12+16
= 28 (molar mass)

NH_3 = ammonia = 14+ 3×1
= 17 (molar mass)

$\frac{\text{molecules of CO}}{\text{avagadro's number}}$ = number of mole

$\frac{48 \times 10^{23}}{6 \times 10^{23}} = 8$ mol

$\frac{\text{molecules of } NH_3}{\text{avagadro's number}}$ = number of mole

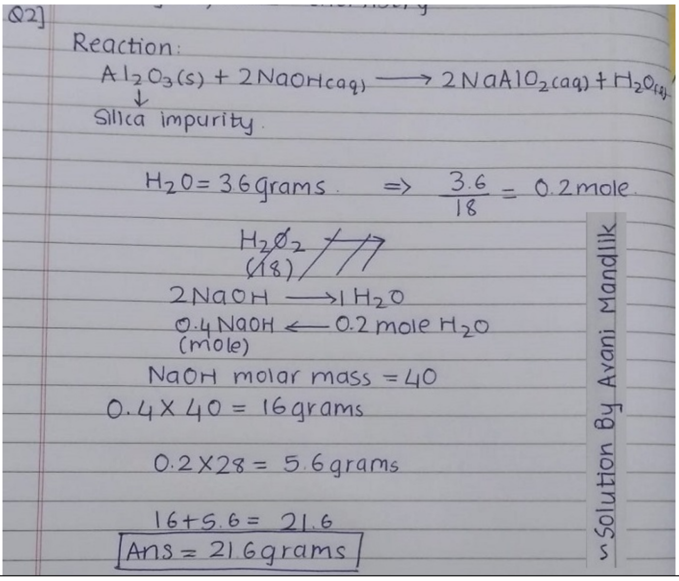
$\frac{12 \times 10^{24}}{6 \times 10^{23}} = 20$ mol

mass of gas = mol × molar mass

mass of CO = 8 × 28 = 224 g
mass of NH_3 = 17×20 = 340g

Difference = 340−224
= $\boxed{116g}$

2



Q2]

Reaction:

$Al_2O_3(s) + 2NaOH(aq) \longrightarrow 2NaAlO_2(aq) + H_2O(l)$
↓
Silica impurity.

H_2O = 3.6 grams. ⇒ $\frac{3.6}{18} = 0.2$ mole.

~~H_2O_2 (18)~~

$2NaOH \longrightarrow 1 H_2O$
$0.4 NaOH \longleftarrow 0.2$ mole H_2O
(mole)

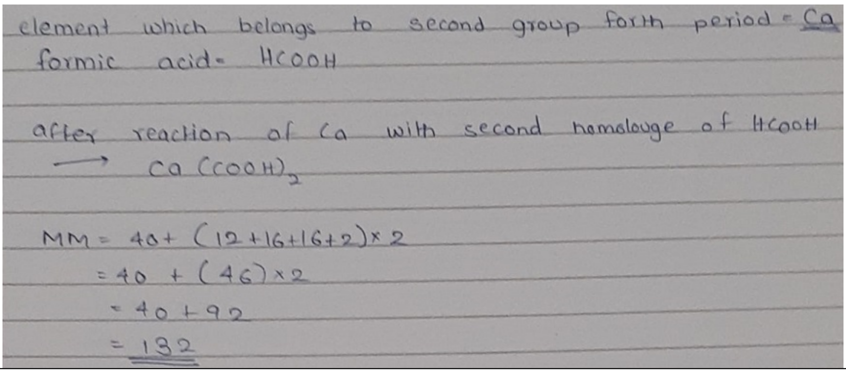
NaOH molar mass = 40
0.4 × 40 = 16 grams

0.2 × 28 = 5.6 grams

16+5.6 = 21.6
$\boxed{\text{Ans} = 21.6 \text{grams}}$

Solution By Avani Mandlik

3

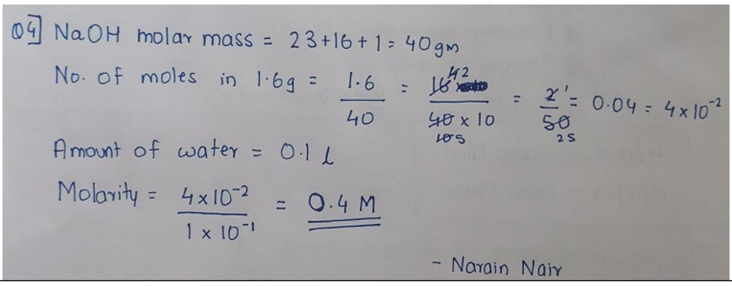


element which belongs to second group forth period = Ca
formic acid = HCOOH

after reaction of Ca with second homolouge of HCOOH
$\longrightarrow$ $Ca(COOH)_2$

$MM = 40 + (12+16+16+2) \times 2$
$= 40 + (46) \times 2$
$= 40 + 92$
$= \underline{\underline{132}}$

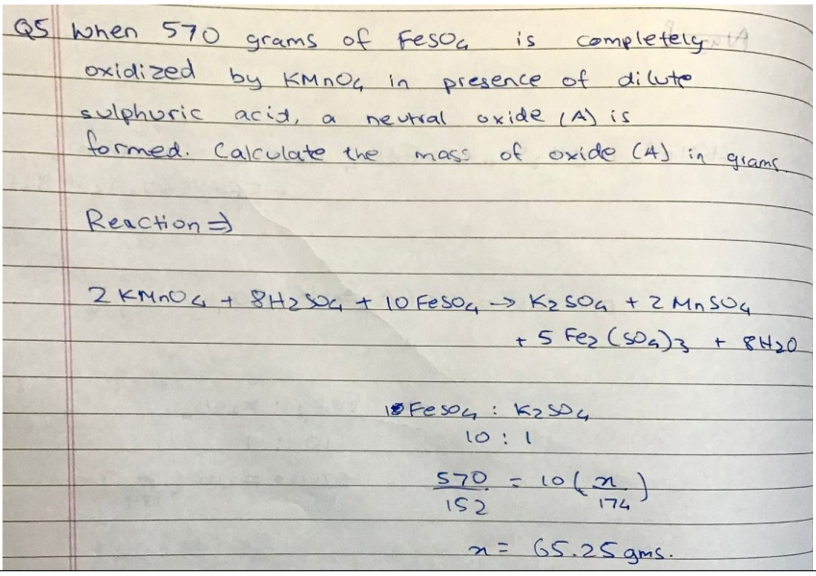
4



Q4] NaOH molar mass = 23+16+1 = 40gm
No. of moles in 1·6g = $\frac{1.6}{40} = \frac{\overset{42}{\cancel{16}}}{\underset{105}{\cancel{40}} \times 10} = \frac{\cancel{2}'}{\underset{25}{\cancel{50}}} = 0.04 = 4 \times 10^{-2}$
Amount of water = 0·1 L
Molarity = $\frac{4 \times 10^{-2}}{1 \times 10^{-1}} = \underline{\underline{0.4\ M}}$

- Narain Nair

5



Q5 When 570 grams of $FeSO_4$ is completely oxidized by $KMnO_4$ in presence of dilute sulphuric acid, a neutral oxide (A) is formed. Calculate the mass of oxide (A) in grams

Reaction ⇒

$2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \rightarrow K_2SO_4 + 2MnSO_4$
$+ 5Fe_2(SO_4)_3 + 8H_2O$

$FeSO_4 : K_2SO_4$
10 : 1

$\frac{570}{152} = 10\left(\frac{x}{174}\right)$

$x = 65.25$ gms.

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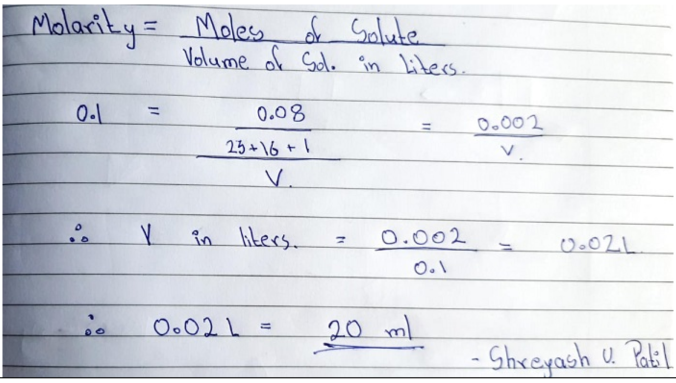


Molarity = $\frac{\text{Moles of Solute}}{\text{Volume of Sol. in liters.}}$

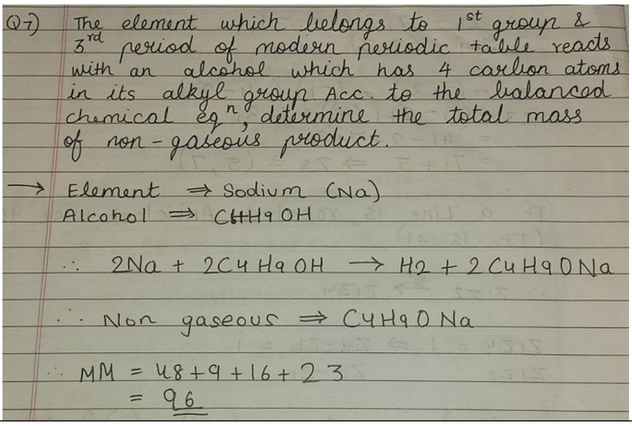
$0.1 = \frac{\frac{0.08}{23+16+1}}{V} = \frac{0.002}{V}$

∴ V in liters. = $\frac{0.002}{0.1}$ = 0.02L

∴ 0.02L = $\underline{20 \text{ ml}}$

- Shreyash U. Patil

7



Q7) The element which belongs to 1st group & 3rd period of modern periodic table reacts with an alcohol which has 4 carbon atoms in its alkyl group. Acc. to the balanced chemical eqn, determine the total mass of non-gaseous product.

→ Element ⇒ Sodium (Na)

Alcohol ⇒ C_4H_9OH

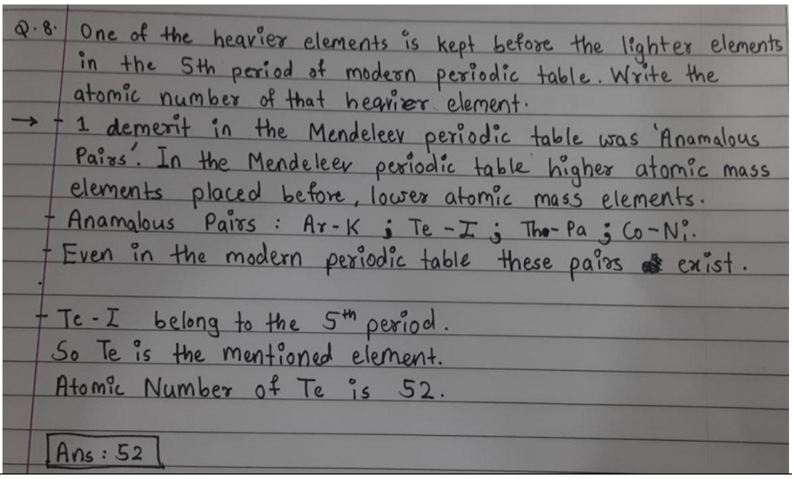
∴ $2Na + 2C_4H_9OH \rightarrow H_2 + 2C_4H_9ONa$

∴ Non gaseous ⇒ C_4H_9ONa

∴ MM = 48+9+16+23

= $\underline{96}$

8



Q.8. One of the heavier elements is kept before the lighter elements in the 5th period of modern periodic table. Write the atomic number of that heavier element.

→ 1 demerit in the Mendeleev periodic table was 'Anamalous Pairs'. In the Mendeleev periodic table higher atomic mass elements placed before, lower atomic mass elements.

Anamalous Pairs : Ar-K ; Te-I ; Th-Pa ; Co-Ni.

Even in the modern periodic table these pairs exist.

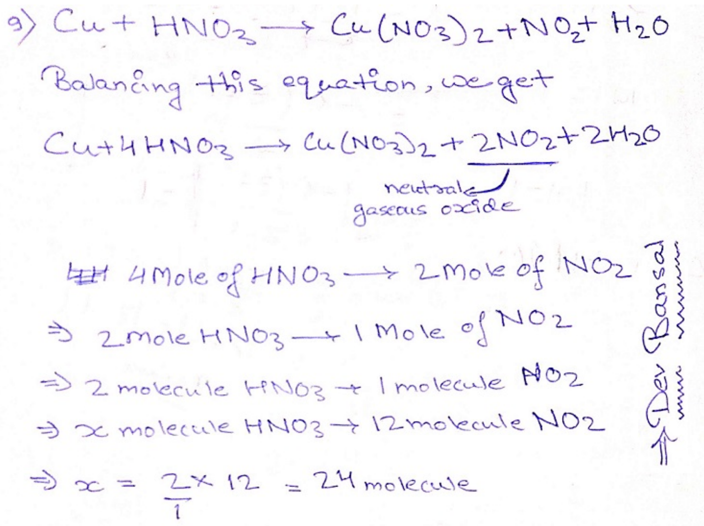
Te-I belong to the 5th period.

So Te is the mentioned element.

Atomic Number of Te is 52.

$\boxed{\text{Ans : 52}}$

9



9) $Cu + HNO_3 \rightarrow Cu(NO_3)_2 + NO_2 + H_2O$

Balancing this equation, we get

$Cu + 4HNO_3 \rightarrow Cu(NO_3)_2 + \underbrace{2NO_2}_{\text{neutral gaseous oxide}} + 2H_2O$

4 Mole of $HNO_3 \rightarrow$ 2 Mole of NO_2

⇒ 2 mole $HNO_3 \rightarrow$ 1 Mole of NO_2

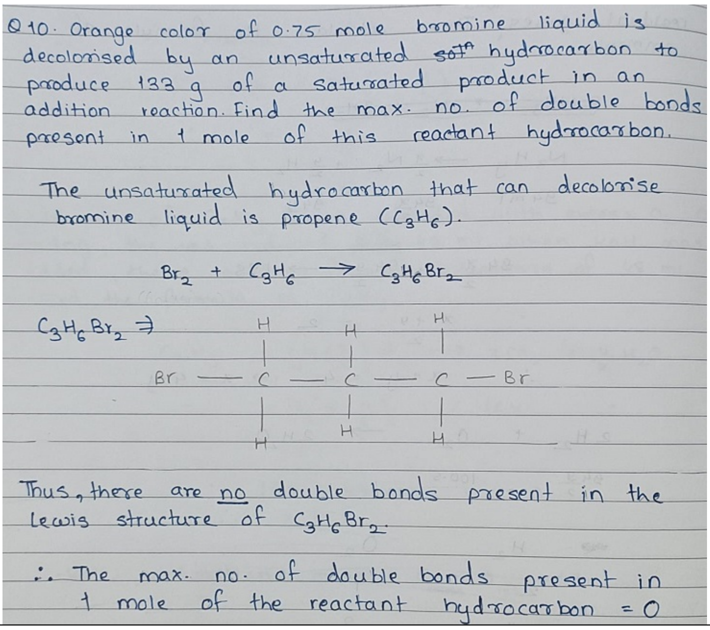
⇒ 2 molecule $HNO_3 \rightarrow$ 1 molecule NO_2

⇒ x molecule $HNO_3 \rightarrow$ 12 molecule NO_2

⇒ $x = \frac{2 \times 12}{1} = 24$ molecule

⇒ Dev Bansal

10



Q.10. Orange color of 0.75 mole bromine liquid is decolorised by an unsaturated ~~sat~~ hydrocarbon to produce 133 g of a saturated product in an addition reaction. Find the max. no. of double bonds present in 1 mole of this reactant hydrocarbon.

The unsaturated hydrocarbon that can decolorise bromine liquid is propene (C_3H_6).

$$Br_2 + C_3H_6 \longrightarrow C_3H_6Br_2$$

$C_3H_6Br_2 \Rightarrow$

Thus, there are no double bonds present in the Lewis structure of $C_3H_6Br_2$.

∴ The max. no. of double bonds present in 1 mole of the reactant hydrocarbon = 0

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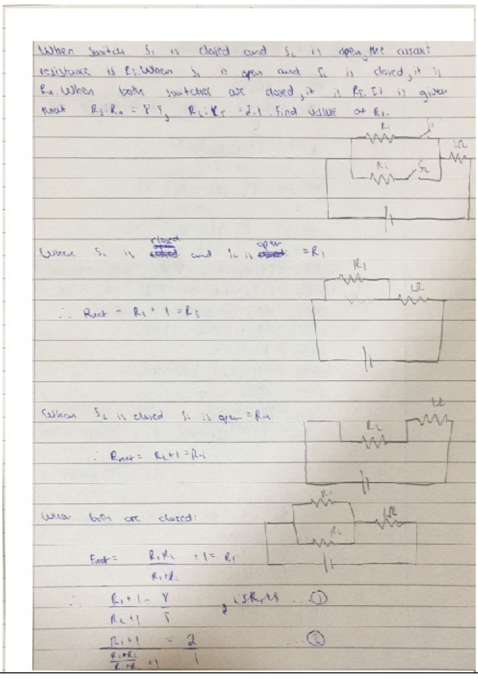


When switch S_1 is closed and S_2 is open, the current resistance is R_3. When S_1 is open and S_2 is closed, it is R_4. When both switches are closed, it is R_5. It is given that $R_3 : R_4 = 8:5$, $R_3 : R_5 = 2:1$. Find value of R_1.

When S_1 is ~~open~~ ~~closed~~ closed and S_2 is ~~closed~~ open $= R_3$

$\therefore R_{net} = R_1 + 1 = R_3$

When S_2 is closed S_1 is open $= R_4$

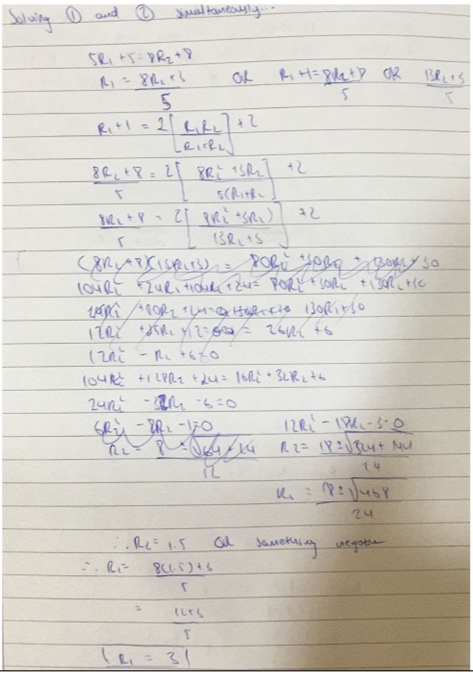
$\therefore R_{net} = R_2 + 1 = R_4$

When both are closed:

$R_{net} = \frac{R_1R_2}{R_1+R_2} + 1 = R_5$

$\therefore \frac{R_1+1}{R_2+1} = \frac{8}{5}$, $5R_1+5$ ... ①

$\frac{R_1+1}{\frac{R_1R_2}{R_1+R_2}+1} = \frac{2}{1}$ ... ②



Solving ① and ② simultaneously...

$5R_1 + 5 = 8R_2 + 8$

$R_1 = \frac{8R_2+3}{5}$ OR $R_1+1 = \frac{8R_2+8}{5}$ OR $\frac{13R_2+3}{5}$

$R_1 + 1 = 2\left[\frac{R_1R_2}{R_1+R_2}\right] + 2$

$\frac{8R_2+8}{5} = 2\left[\frac{8R_2^2+3R_2}{5(R_1+R_2)}\right] + 2$

$\frac{8R_2+8}{5} = 2\left[\frac{8R_2^2+3R_2)}{13R_2+3}\right] + 2$

~~$(8R_2+8)(13R_2+3) = 80R_2^2 + 30R_2 + 130R_2 + 30$~~

$104R_2^2 + 24R_2 + 104R_2 + 24 = 80R_2^2 + 30R_2 + 130R_2 + 10$

~~$144R_2^2 + 10R_2 + 24 =$ [illegible] $130R_2 + 30$~~

~~$12R_2^2 + 25R_2 + 12 =$ [illegible] $= 26R_2 + 6$~~

$12R_2^2 - R_2 + 6 = 0$

$104R_2^2 + 128R_2 + 24 = 16R_2^2 + 32R_2 + 6$

$24R_2^2 - 3R_2 - 6 = 0$

~~$6R_2^2 - 8R_2 - 1 = 0$~~ $\quad 12R_2^2 - 18R_2 - 3 = 0$

~~$R_2 = \frac{8 \pm \sqrt{64 + 24}}{12}$~~ $\quad R_2 = \frac{18 \pm \sqrt{324 + 144}}{24}$

$R_2 = \frac{18 \pm \sqrt{468}}{24}$

$\therefore R_2 = 1.5$ or something negative

$\therefore R_1 = \frac{8(1.5)+3}{5}$

$= \frac{12+3}{5}$

$\boxed{R_1 = 3}$

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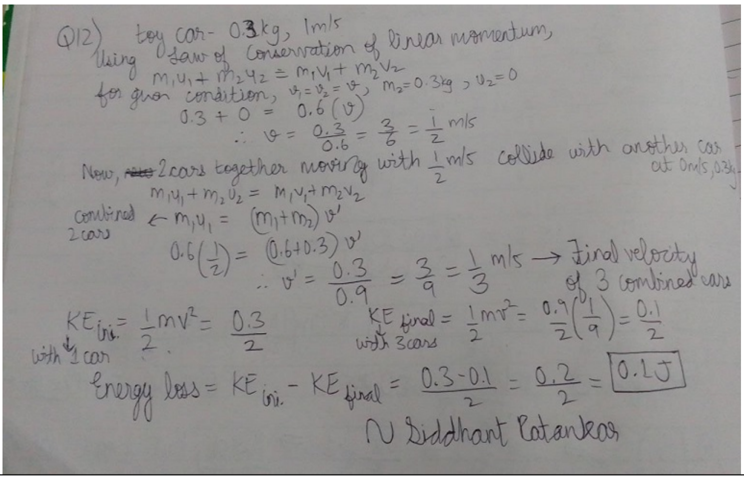

Q12) toy car - 0.3kg, 1m/s

Using law of Conservation of linear momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

for given condition, $v_1 = v_2 = v$, $m_2 = 0.3kg$, $u_2 = 0$

$0.3 + 0 = 0.6(v)$

$\therefore v = \frac{0.3}{0.6} = \frac{3}{6} = \frac{1}{2}$ m/s

Now, 2cars together moving with $\frac{1}{2}$ m/s collide with another car at 0m/s, 0.3kg

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

combined 2cars $\leftarrow m_1u_1 = (m_1+m_2)v'$

$0.6\left(\frac{1}{2}\right) = (0.6+0.3)v'$

$\therefore v' = \frac{0.3}{0.9} = \frac{3}{9} = \frac{1}{3}$ m/s → final velocity of 3 combined cars

$KE_{ini} = \frac{1}{2}mv^2 = \frac{0.3}{2}$ with 1 car

$KE_{final} = \frac{1}{2}mv^2 = \frac{0.9}{2}\left(\frac{1}{9}\right) = \frac{0.1}{2}$ with 3cars

Energy loss $= KE_{ini} - KE_{final} = \frac{0.3-0.1}{2} = \frac{0.2}{2} = \boxed{0.1J}$

~ Siddhant Patankar

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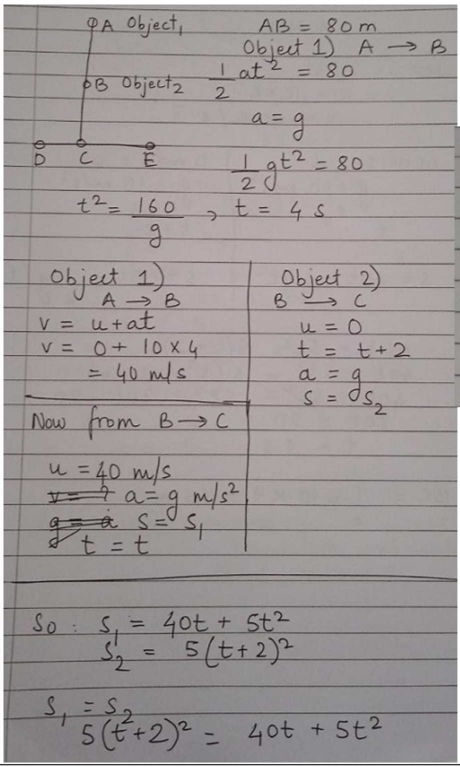


A Object$_1$

B Object$_2$

D C E

AB = 80 m

Object 1) A → B

$\frac{1}{2}at^2 = 80$

$a = g$

$\frac{1}{2}gt^2 = 80$

$t^2 = \frac{160}{g}$, $t = 4$ s

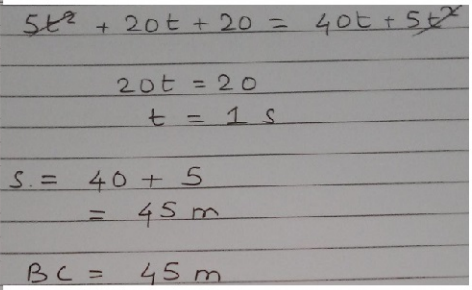
Object 1) A → B	Object 2) B → C
$v = u + at$ $v = 0 + 10 \times 4$ $= 40$ m/s	$u = 0$ $t = t + 2$ $a = g$ $s = s_2$
Now from B → C $u = 40$ m/s ~~v = ?~~ $a = g$ m/s^2 ~~g = a~~ $S = S_1$ $t = t$	

So : $S_1 = 40t + 5t^2$

$S_2 = 5(t+2)^2$

$S_1 = S_2$

$5(t+2)^2 = 40t + 5t^2$



$\cancel{5t^2} + 20t + 20 = 40t + \cancel{5t^2}$

$20t = 20$

$t = 1$ s

$S = 40 + 5$

$= 45$ m

BC = 45 m

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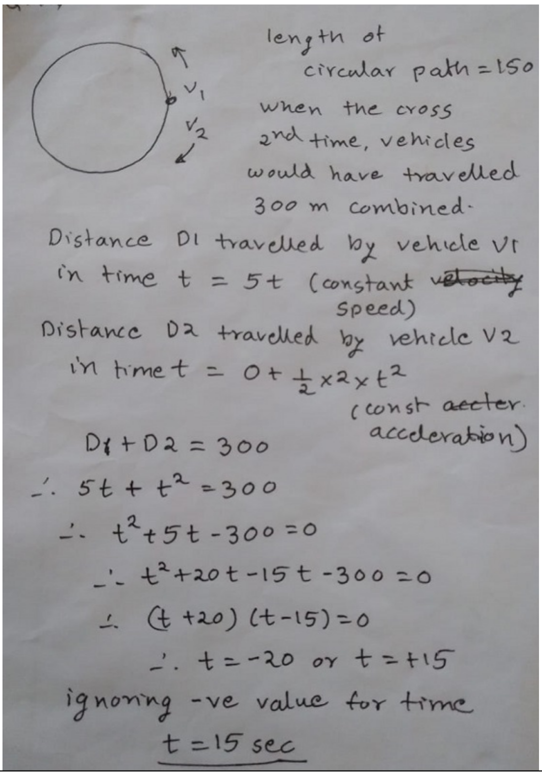


v_1

v_2

length of circular path = 150

when the cross 2nd time, vehicles would have travelled 300 m combined.

Distance D1 travelled by vehicle V1 in time t = 5t (constant ~~velocity~~ speed)

Distance D2 travelled by vehicle V2 in time t = $0 + \frac{1}{2} \times 2 \times t^2$ (const ~~acceler.~~ acceleration)

$D1 + D2 = 300$

$\therefore 5t + t^2 = 300$

$\therefore t^2 + 5t - 300 = 0$

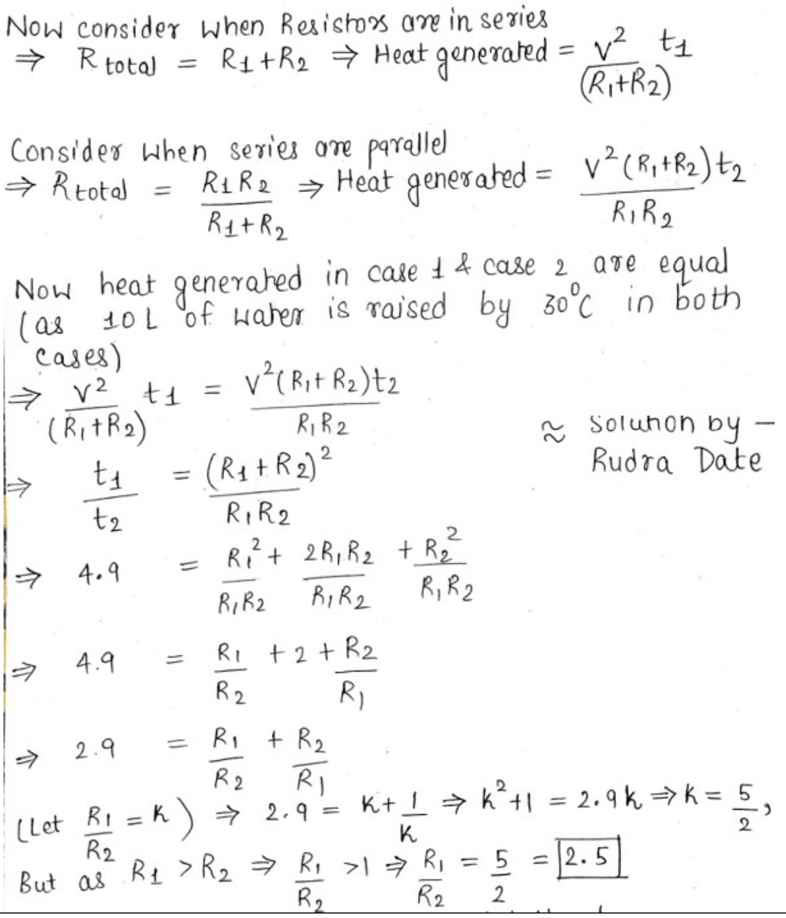
$\therefore t^2 + 20t - 15t - 300 = 0$

$\therefore (t+20)(t-15) = 0$

$\therefore t = -20$ or $t = +15$

ignoring -ve value for time

$\underline{t = 15 \text{ sec}}$

15 

Now consider when Resistors are in series

$\Rightarrow R_{total} = R_1+R_2 \Rightarrow$ Heat generated $= \frac{V^2}{(R_1+R_2)} t_1$

Consider when series are parallel

$\Rightarrow R_{total} = \frac{R_1R_2}{R_1+R_2} \Rightarrow$ Heat generated $= \frac{V^2(R_1+R_2)t_2}{R_1R_2}$

Now heat generated in case 1 & case 2 are equal (as 10 L of water is raised by 30^0C in both cases)

$$\Rightarrow \frac{V^2}{(R_1+R_2)} t_1 = \frac{V^2(R_1+R_2)t_2}{R_1R_2}$$

≈ solution by – Rudra Date

$$\Rightarrow \frac{t_1}{t_2} = \frac{(R_1+R_2)^2}{R_1R_2}$$

$$\Rightarrow 4.9 = \frac{R_1^2}{R_1R_2} + \frac{2R_1R_2}{R_1R_2} + \frac{R_2^2}{R_1R_2}$$

$$\Rightarrow 4.9 = \frac{R_1}{R_2} + 2 + \frac{R_2}{R_1}$$

$$\Rightarrow 2.9 = \frac{R_1}{R_2} + \frac{R_2}{R_1}$$

(Let $\frac{R_1}{R_2} = k$) $\Rightarrow 2.9 = k + \frac{1}{k} \Rightarrow k^2+1 = 2.9k \Rightarrow k = \frac{5}{2}$,

But as $R_1 > R_2 \Rightarrow \frac{R_1}{R_2} > 1 \Rightarrow \frac{R_1}{R_2} = \frac{5}{2} = \boxed{2.5}$

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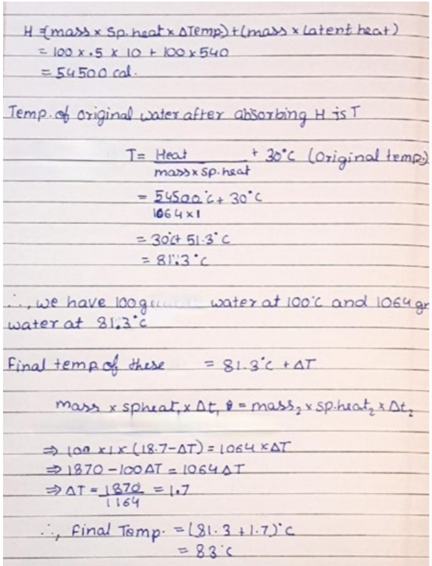
16 

H = (mass × Sp. heat × ΔTemp) + (mass × Latent heat)

$= 100 \times .5 \times 10 + 100 \times 540$

$= 54500$ cal.

Temp. of original water after absorbing H is T

$T = \frac{Heat}{mass \times sp.heat} + 30°C$ (Original temp.)

$= \frac{54500}{1064 \times 1}°C + 30°C$

$= 30°C + 51.3°C$

$= 81.3°C$

∴, we have 100gm water at 100°C and 1064 gr water at 81.3°C

Final temp of these $= 81.3°C + \Delta T$

$mass \times spheat_1 \times \Delta t_1 = mass_2 \times sp.heat_2 \times \Delta t_2$

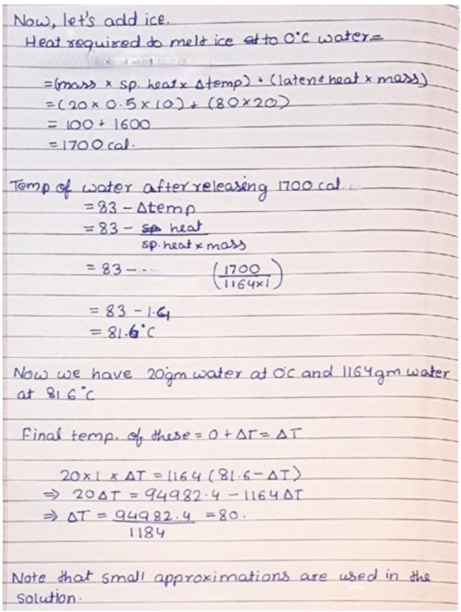
$\Rightarrow 100 \times 1 \times (18.7 - \Delta T) = 1064 \times \Delta T$

$\Rightarrow 1870 - 100\Delta T = 1064 \Delta T$

$\Rightarrow \Delta T = \frac{1870}{1164} = 1.7$

∴, Final Temp. $= (81.3 + 1.7)°C$

$= 83°C$



Now, let's add ice.

Heat required to melt ice ~~at~~ to 0°C water =

$= (mass \times sp.heat \times \Delta temp) + (latent heat \times mass)$

$= (20 \times 0.5 \times 10) + (80 \times 20)$

$= 100 + 1600$

$= 1700$ cal.

Temp of water after releasing 1700 cal.

$= 83 - \Delta temp$

$= 83 - \frac{heat}{sp.heat \times mass}$

$= 83 - \left(\frac{1700}{1164 \times 1}\right)$

$= 83 - 1.6$

$= 81.6°C$

Now we have 20gm water at 0°C and 1164gm water at 81.6°C

Final temp. of these $= 0 + \Delta T = \Delta T$

$20 \times 1 \times \Delta T = 1164(81.6 - \Delta T)$

$\Rightarrow 20\Delta T = 94982.4 - 1164\Delta T$

$\Rightarrow \Delta T = \frac{94982.4}{1184} = 80.$

Note that small approximations are used in the solution.

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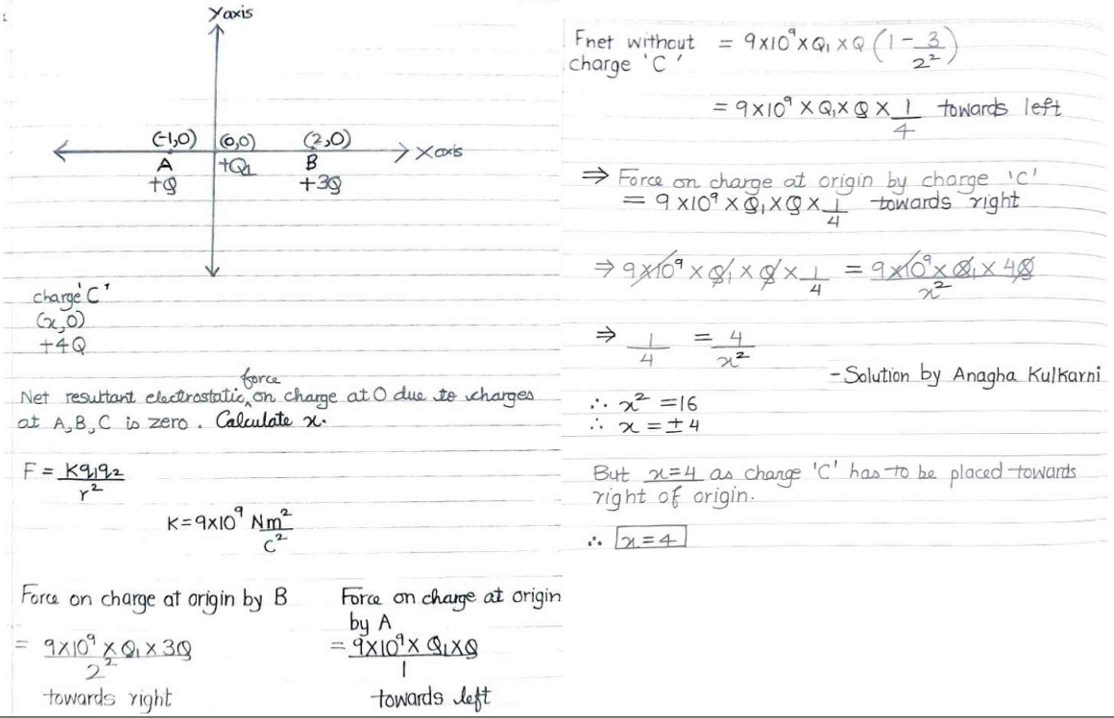

charge 'C'
(x,0)
+4Q

Net resultant electrostatic force on charge at O due to charges at A, B, C is zero. Calculate x.

$F = \frac{Kq_1q_2}{r^2}$

$K = 9\times10^9 \frac{Nm^2}{C^2}$

Force on charge at origin by B

$= \frac{9\times10^9 \times Q_1 \times 3Q}{2^2}$

towards right

Force on charge at origin by A

$= \frac{9\times10^9 \times Q_1 \times Q}{1}$

towards left

Fnet without charge 'C' $= 9\times10^9 \times Q_1 \times Q \left(1 - \frac{3}{2^2}\right)$

$= 9\times10^9 \times Q_1 \times Q \times \frac{1}{4}$ towards left

⇒ Force on charge at origin by charge 'C'
$= 9\times10^9 \times Q_1 \times Q \times \frac{1}{4}$ towards right

$\Rightarrow \cancel{9\times10^9} \times \cancel{Q_1} \times \cancel{Q} \times \frac{1}{4} = \frac{\cancel{9\times10^9} \times \cancel{Q_1} \times 4\cancel{Q}}{x^2}$

$\Rightarrow \frac{1}{4} = \frac{4}{x^2}$

– Solution by Anagha Kulkarni

$\therefore x^2 = 16$
$\therefore x = \pm 4$

But $x=4$ as charge 'C' has to be placed towards right of origin.

$\therefore \boxed{x=4}$

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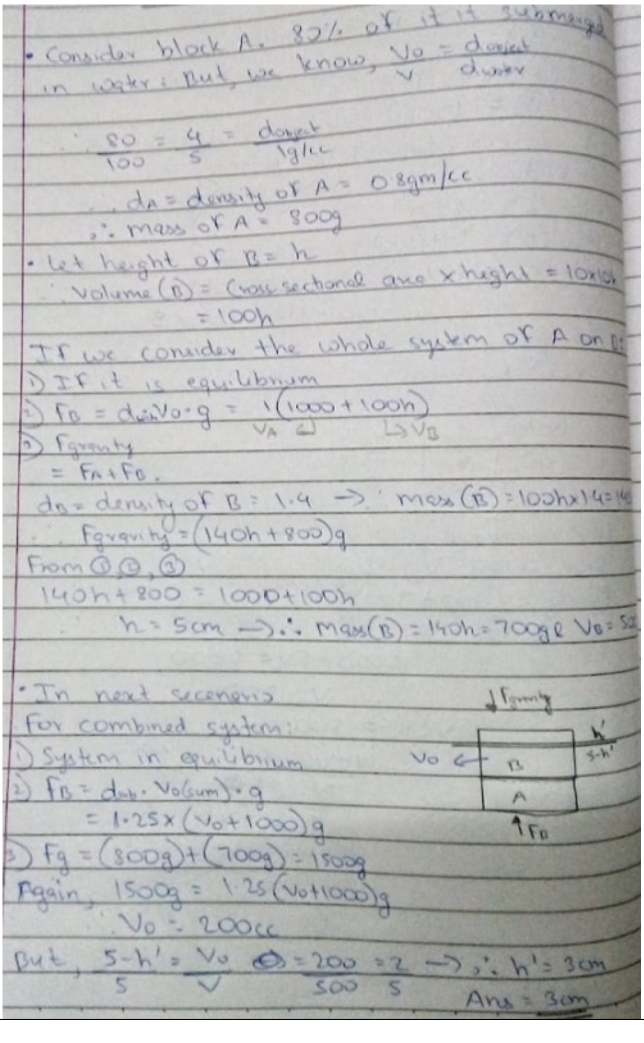

• Consider block A. 80% of it is submerged in water. But, we know, $\frac{V_o}{V} = \frac{d_{object}}{d_{water}}$

$\therefore \frac{80}{100} = \frac{4}{5} = \frac{d_{object}}{1g/cc}$

$\therefore d_A$ = density of A = 0.8gm/cc
∴ mass of A = 800g

• Let height of B = h
∴ volume (B) = Cross sectional area × height = 10×10
= 100h

If we consider the whole system of A on B:
1) If it is equilibrium
2) $F_B = d_w V_o \cdot g = 1(\underset{V_A}{1000} + \underset{V_B}{100h})$
3) $F_{gravity}$
$= F_A + F_B$.

d_B = density of B = 1.4 → ∴ mass (B) = 100h×1.4 = 14
∴ $F_{gravity} = (140h + 800)g$

From ①, ②, ③
140h + 800 = 1000 + 100h
∴ h = 5cm → ∴ mass(B) = 140h = 700g & V_B = 5

• In next scenario
For combined system:
1) System in equilibrium
2) $F_B = d_{sub} \cdot V_o(sum) \cdot g$
$= 1.25 \times (V_o + 1000)g$
3) $F_g = (800g) + (700g) = 1500g$

Again, $1500g = 1.25(V_o+1000)g$
∴ $V_o = 200cc$

But, $\frac{5-h'}{5} = \frac{V_o}{V} = \frac{200}{500} = \frac{2}{5}$ → ∴ h' = 3cm

Ans = 3cm

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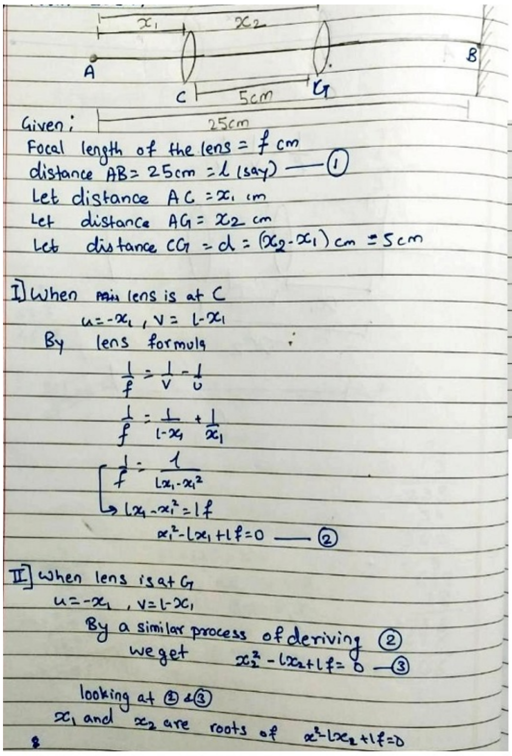


Given;

Focal length of the lens = f cm

distance AB = 25cm = l (say) —— ①

Let distance AC = x_1 cm

Let distance AG = x_2 cm

Let distance CG = d = $(x_2 - x_1)$ cm = 5cm

I] When ~~main~~ lens is at C

$u = -x_1$, $v = l - x_1$

By lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{l-x_1} + \frac{1}{x_1}$$

$$\frac{1}{f} = \frac{1}{lx_1 - x_1^2}$$

$$\to lx_1 - x_1^2 = lf$$

$$x_1^2 - lx_1 + lf = 0 \quad \text{—— ②}$$

II] When lens is at G

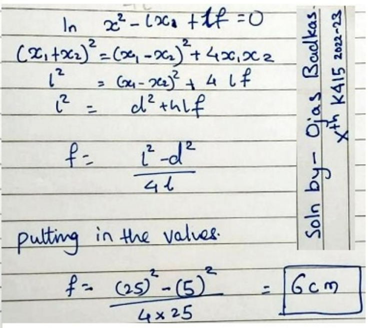
$u = -x_1$, $v = l - x_1$

By a similar process of deriving ②

we get $x_2^2 - lx_2 + lf = 0$ — ③

looking at ② & ③

x_1 and x_2 are roots of $x^2 - lx_2 + lf = 0$



In $x^2 - lx_1 + lf = 0$

$$(x_1 + x_2)^2 = (x_1 - x_2)^2 + 4x_1x_2$$

$$l^2 = (x_1 - x_2)^2 + 4lf$$

$$l^2 = d^2 + 4lf$$

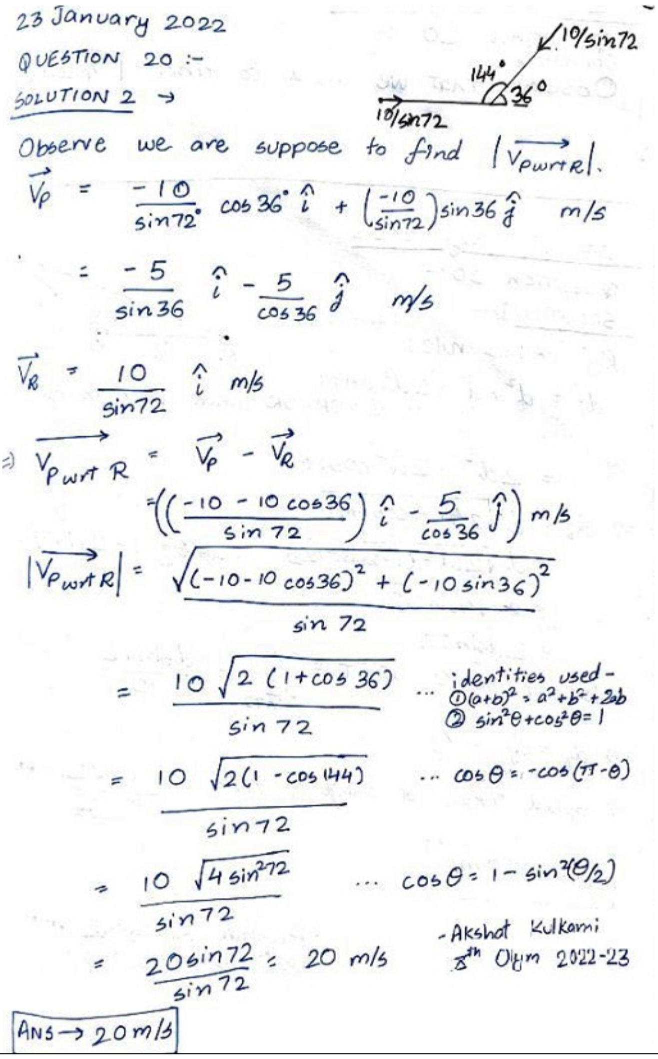
$$f = \frac{l^2 - d^2}{4l}$$

putting in the values.

$$f = \frac{(25)^2 - (5)^2}{4 \times 25} = \boxed{6\,\text{cm}}$$

Soln by – Ojas Badkas
Xth K415 2022-23

20



23 January 2022

QUESTION 20 :-

SOLUTION 2 →

Observe we are suppose to find $|\overrightarrow{V_{P\,wrt\,R}}|$.

$$\vec{V_P} = \frac{-10}{\sin 72°} \cos 36° \hat{i} + \left(\frac{-10}{\sin 72}\right) \sin 36 \hat{j} \quad m/s$$

$$= \frac{-5}{\sin 36} \hat{i} - \frac{5}{\cos 36} \hat{j} \quad m/s$$

$$\vec{V_R} = \frac{10}{\sin 72} \hat{i} \quad m/s$$

$$\Rightarrow \overrightarrow{V_{P\,wrt\,R}} = \vec{V_P} - \vec{V_R}$$

$$= \left(\left(\frac{-10 - 10\cos 36}{\sin 72}\right)\hat{i} - \frac{5}{\cos 36}\hat{j}\right) m/s$$

$$|\overrightarrow{V_{P\,wrt\,R}}| = \frac{\sqrt{(-10 - 10\cos 36)^2 + (-10\sin 36)^2}}{\sin 72}$$

$$= \frac{10\sqrt{2(1+\cos 36)}}{\sin 72} \quad \ldots \text{identities used -}$$

① $(a+b)^2 = a^2 + b^2 + 2ab$
② $\sin^2\theta + \cos^2\theta = 1$

$$= \frac{10\sqrt{2(1-\cos 144)}}{\sin 72} \quad \ldots \cos\theta = -\cos(\pi - \theta)$$

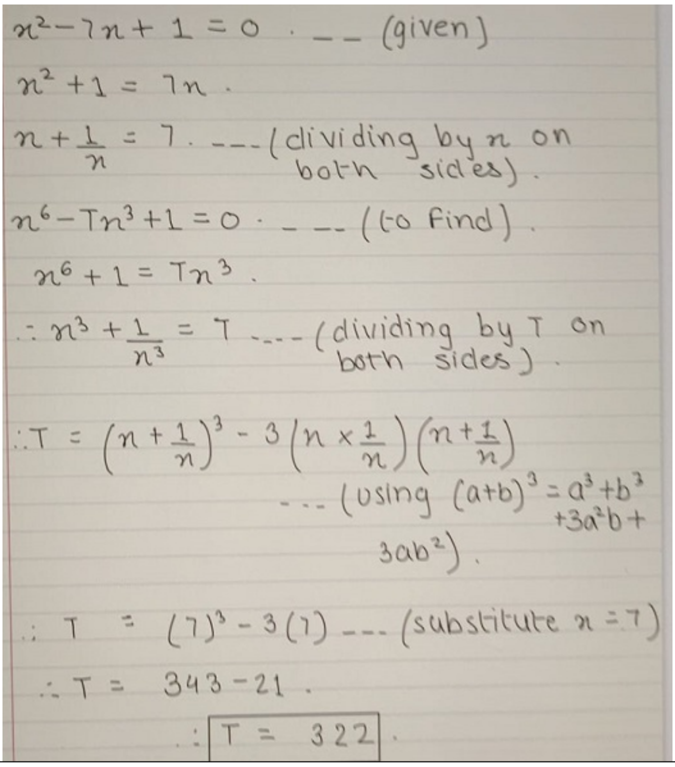
$$= \frac{10\sqrt{4\sin^2 72}}{\sin 72} \quad \ldots \cos\theta = 1 - \sin^2(\theta/2)$$

$$= \frac{20\sin 72}{\sin 72} = 20 \; m/s$$

- Akshat Kulkarni
8th Olym 2022-23

$\boxed{\text{ANS} \to 20\,m/s}$

21



$x^2 - 7x + 1 = 0$. – – (given)

$x^2 + 1 = 7x$.

$x + \frac{1}{x} = 7$. – – – (dividing by x on both sides) .

$x^6 - Tx^3 + 1 = 0$. – – – (to find) .

$x^6 + 1 = Tx^3$.

$\therefore x^3 + \frac{1}{x^3} = T$ – – – – (dividing by T on both sides) .

$\therefore T = \left(x + \frac{1}{x}\right)^3 - 3\left(x \times \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

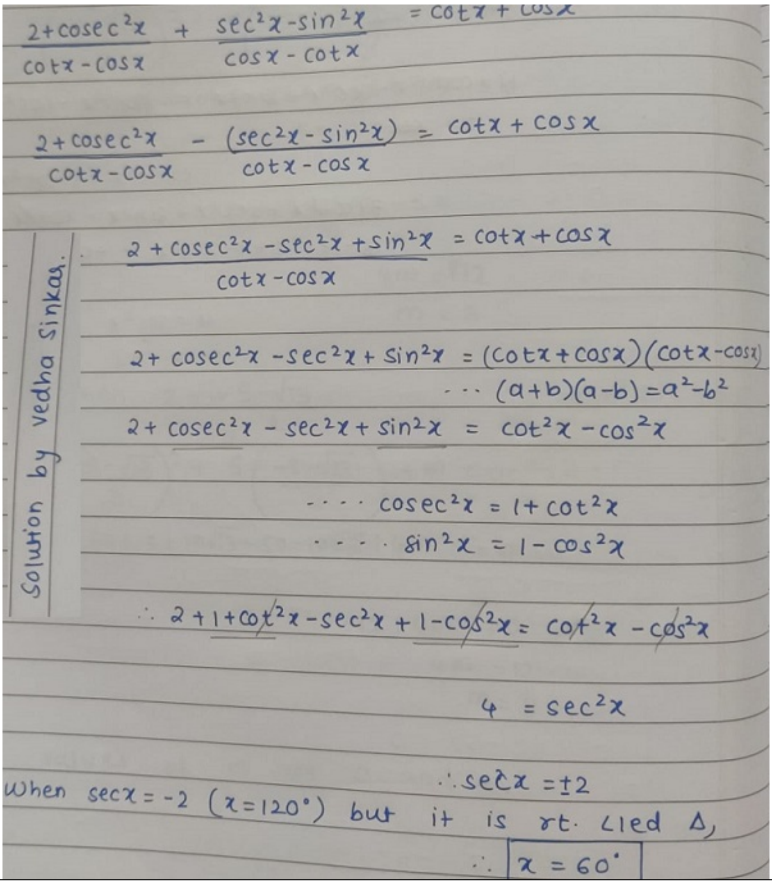
– – – (using $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$) .

$\therefore T = (7)^3 - 3(7)$ – – – (substitute $x = 7$)

$\therefore T = 343 - 21$.

$\therefore \boxed{T = 322}$.

22



$\frac{2 + \operatorname{cosec}^2 x}{\cot x - \cos x} + \frac{\sec^2 x - \sin^2 x}{\cos x - \cot x} = \cot x + \cos x$

$\frac{2 + \operatorname{cosec}^2 x}{\cot x - \cos x} - \frac{(\sec^2 x - \sin^2 x)}{\cot x - \cos x} = \cot x + \cos x$

Solution by vedha Sinkar.

$\therefore \frac{2 + \operatorname{cosec}^2 x - \sec^2 x + \sin^2 x}{\cot x - \cos x} = \cot x + \cos x$

$2 + \operatorname{cosec}^2 x - \sec^2 x + \sin^2 x = (\cot x + \cos x)(\cot x - \cos x)$

· · · $(a+b)(a-b) = a^2 - b^2$

$2 + \underline{\operatorname{cosec}^2 x} - \sec^2 x + \underline{\sin^2 x} = \cot^2 x - \cos^2 x$

· · · · $\operatorname{cosec}^2 x = 1 + \cot^2 x$

· · · · $\sin^2 x = 1 - \cos^2 x$

$\therefore 2 + \underline{1 + \cancel{\cot^2 x}} - \sec^2 x + \underline{1 - \cancel{\cos^2 x}} = \cancel{\cot^2 x} - \cancel{\cos^2 x}$

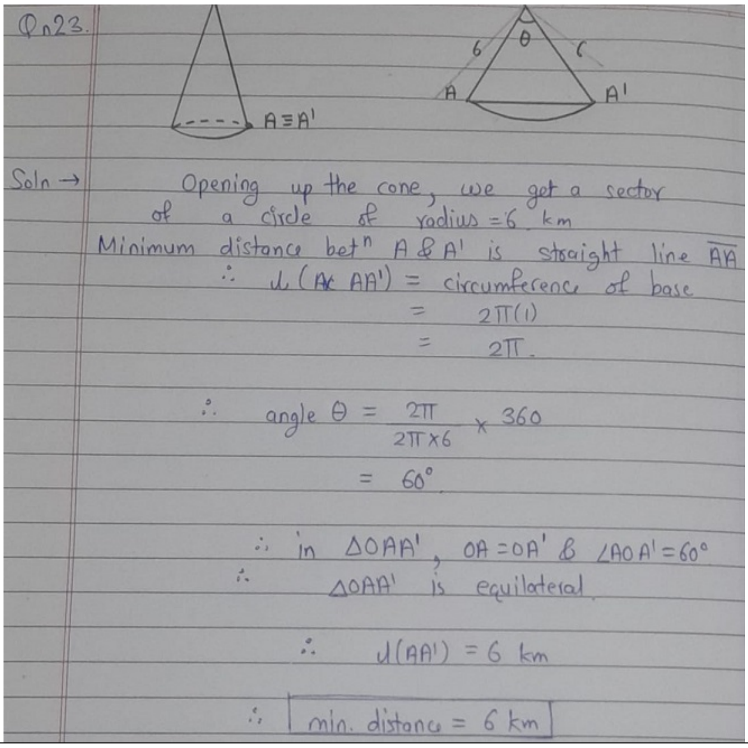
$4 = \sec^2 x$

$\therefore \sec x = \pm 2$

When $\sec x = -2$ ($x = 120°$) but it is rt. ∠led Δ,

$\therefore \boxed{x = 60°}$

23



Qn23.

Soln → Opening up the cone, we get a sector of a circle of radius = 6 km

Minimum distance betn A & A' is straight line $\overline{AA}$

∴ d(~~A~~ AA') = circumference of base

= 2π(1)

= 2π.

∴ angle $\theta = \frac{2\pi}{2\pi \times 6} \times 360$

= 60°.

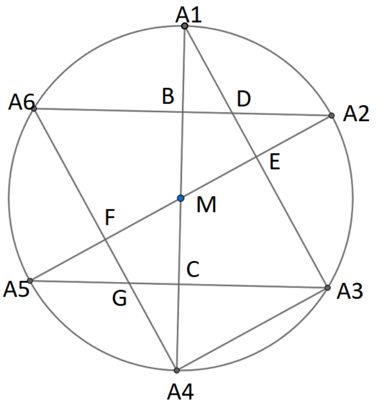
∴ in ΔOAA', OA = OA' & ∠AOA' = 60°

∴ ΔOAA' is equilateral.

∴ d(AA') = 6 km

∴ $\boxed{\text{min. distance} = 6 \text{ km}}$

24



Note:

B is mid point of MA_1 and A_2A_6

C is mid point of MA_4 and A_3A_5

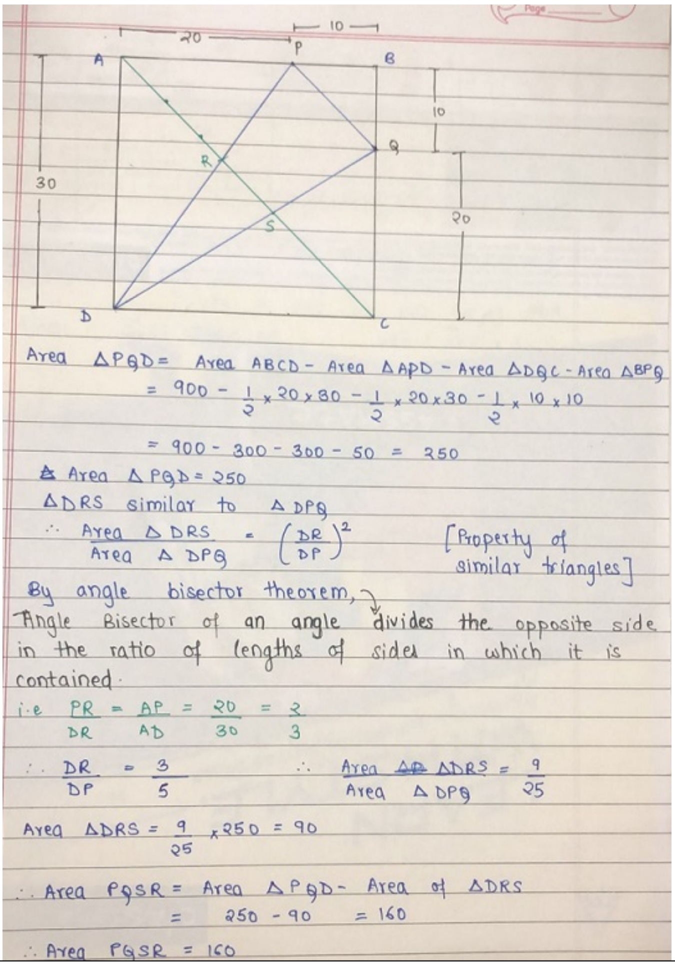
E is mid point of MA_2 and A_1A_3

F is mid point o MA_5 and A_4A_6.

Also all right angled triangles are $30-60-90$. [ABC] means area of triangle ABC.

$$
\begin{aligned}
& A_1A_4 = 24 \Rightarrow A_3A_4 = 12 \text{ and } A_1A_3 = 12\sqrt{3} \\
& \Rightarrow [A_1A_3A_4] = \frac{12 \times 12\sqrt{3}}{2} = 72\sqrt{3} \\
& BA_4 = 18, BA_6 = \frac{18}{\sqrt{3}} \\
& \Rightarrow [BA_4A_6] = 18 \times \frac{18}{\sqrt{3}} \times \frac{1}{2} = 6 \times 9\sqrt{3} = 54\sqrt{3} \\
& A_1B = 6, BD = \frac{6}{\sqrt{3}} \Rightarrow [A_1BD] = 6 \times \frac{6}{\sqrt{3}} \times \frac{1}{2} = 6\sqrt{3} \\
& \therefore [A_1BD] = [A_2DE] = [A_5FG] = [A_4CG] = 6\sqrt{3} \\
& CA_4 = 6; CA_3 = 6\sqrt{3} \Rightarrow [A_3A_4C] = 6/2 \times 6\sqrt{3} = 18\sqrt{3} \\
& \text{ Required area } = [A_1A_3A_4] + [A_4A_6B] + 2\,[A_2DE] \\
& = 72\sqrt{3} + 54\sqrt{3} + 2(6\sqrt{3}) \\
& = 138\sqrt{3} \\
& K = 138
\end{aligned}
$$

25



Area ΔPQD = Area $ABCD$ − Area ΔAPD − Area ΔDQC − Area ΔBPQ

$= 900 - \frac{1}{2} \times 20 \times 30 - \frac{1}{2} \times 20 \times 30 - \frac{1}{2} \times 10 \times 10$

$= 900 - 300 - 300 - 50 = 250$

~~Δ~~ Area $\Delta PQD = 250$

ΔDRS similar to ΔDPQ

$\therefore \frac{\text{Area } \Delta DRS}{\text{Area } \Delta DPQ} = \left(\frac{DR}{DP}\right)^2$ [Property of similar triangles]

By angle bisector theorem,

Angle Bisector of an angle divides the opposite side in the ratio of lengths of sides in which it is contained.

i.e $\frac{PR}{DR} = \frac{AP}{AD} = \frac{20}{30} = \frac{2}{3}$

$\therefore \frac{DR}{DP} = \frac{3}{5}$ $\therefore \frac{\text{Area } \cancel{\Delta P}\, \Delta DRS}{\text{Area } \Delta DPQ} = \frac{9}{25}$

Area $\Delta DRS = \frac{9}{25} \times 250 = 90$

$\therefore$ Area $PQSR$ = Area ΔPQD − Area of ΔDRS

$= 250 - 90 = 160$

$\therefore$ Area $PQSR = 160$

26 For $x \neq \frac{9}{4}, f(x)$ is defined as

$$f(x) = \frac{8x\sqrt{x} + 3 - 2\sqrt{x} - 12x}{2\sqrt{x} - 3}$$

Find $f(9) + f(10) + f(11) + \cdots + f(16)$

Let $\sqrt{x} = a \Rightarrow x = a^2$

$$\therefore f(x) = \frac{8 \times a \times a^2 + 3 - 2 \times a - 12 \times a^2}{2 \times a - 3} = \frac{8a^3 - 12a^2 - 2a + 3}{2a - 3}$$

$$8a^3 - 12a^2 - 2a + 3 = (2a - 3) \times (4a^2 - 1)$$

$$\Rightarrow f(x) = \frac{(2a - 3) \times (4a^2 - 1)}{2a - 3} = 4a^2 - 1$$

$\because a^2 = x,$

$f(x) = 4x - 1$

$\therefore f(9) + f(10) + f(11) + \cdots + f(16)$

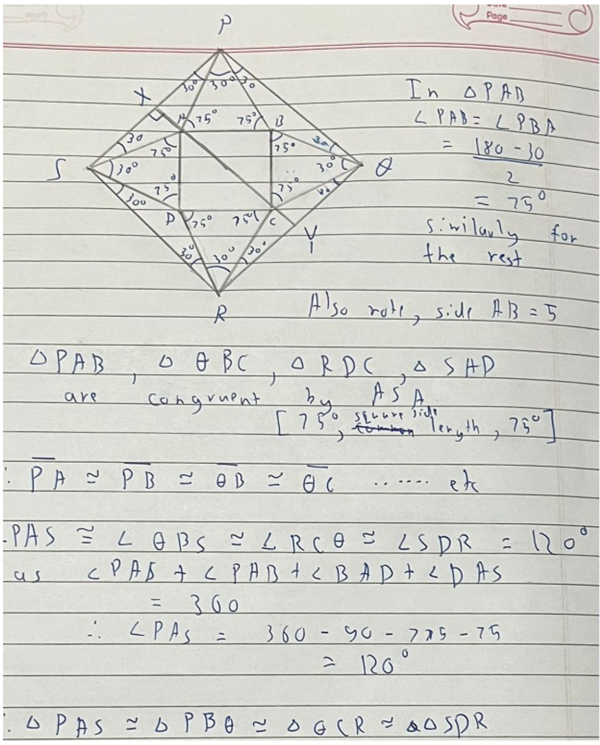
$= (4 \times 9 - 1) + (4 \times 10 - 1) + (4 \times 11 - 1) + \cdots + (4 \times 16 - 1)$

$= 4 \times 9 + 4 \times 10 + 4 \times 11 + \cdots + 4 \times 16 + (8) \times (-1)$

$= 4(9 + 10 + 11 + 12 + 13 + 14 + 15 + 16) - 8$

$= 400 - 8 = \boxed{\mathbf{392}}$

27



In $\triangle PAB$
$\angle PAB = \angle PBA$
$= \frac{180 - 30}{2}$
$= 75°$
Similarly for the rest

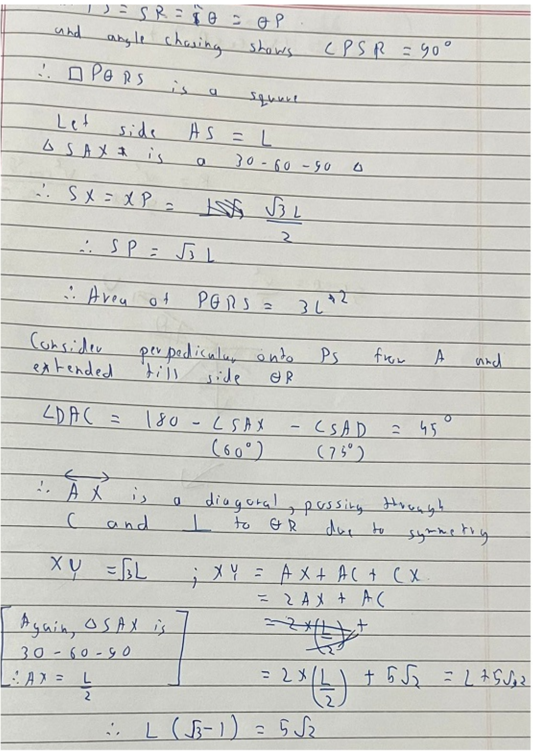
Also note, side $AB = 5$

$\triangle PAB$, $\triangle \theta BC$, $\triangle RDC$, $\triangle SAD$ are congruent by ASA
[75°, ~~common~~ square side length, 75°]

$\therefore \overline{PA} \simeq \overline{PB} \simeq \overline{\theta B} \simeq \overline{\theta C}$ ...... etc

$\angle PAS \cong \angle \theta BS \simeq \angle RC\theta \simeq \angle SDR = 120°$
as $\angle PAS + \angle PAB + \angle BAD + \angle DAS$
$= 360$
$\therefore \angle PAS = 360 - 90 - 75 - 75$
$= 120°$

$\therefore \triangle PAS \simeq \triangle PB\theta \simeq \triangle \theta CR \simeq \triangle SDR$



$= SR = R\theta = \theta P$
and angle chasing shows $\angle PSR = 90°$
$\therefore \square P\theta RS$ is a square

Let side $AS = L$
$\triangle SAX$ is a 30-60-90 $\triangle$
$\therefore SX = XP = \frac{\sqrt{3}L}{2}$
$\therefore SP = \sqrt{3}L$

$\therefore$ Area of $P\theta RS = 3L^2$

Consider perpendicular onto PS from A and extended till side θR

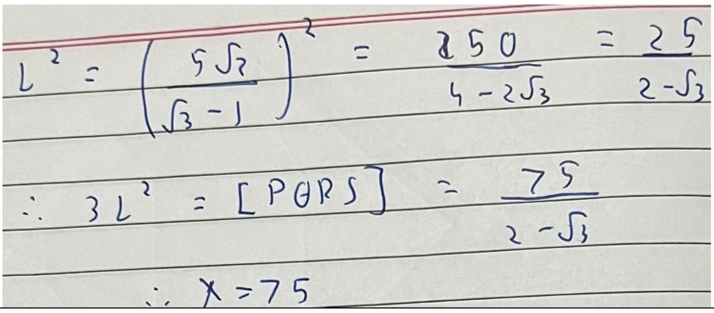
$\angle DAC = 180 - \underset{(60°)}{\angle SAX} - \underset{(75°)}{\angle SAD} = 45°$

$\therefore \overleftrightarrow{AX}$ is a diagonal, passing through C and $\perp$ to θR due to symmetry

$XY = \sqrt{3}L$; $XY = AX + AC + CX$
$= 2AX + AC$
~~$= 2 \times \left(\frac{L}{2}\right) +$~~
$= 2 \times \left(\frac{L}{2}\right) + 5\sqrt{2} = L + 5\sqrt{2}$

[Again, $\triangle SAX$ is 30-60-90
$\therefore AX = \frac{L}{2}$]

$\therefore L(\sqrt{3} - 1) = 5\sqrt{2}$

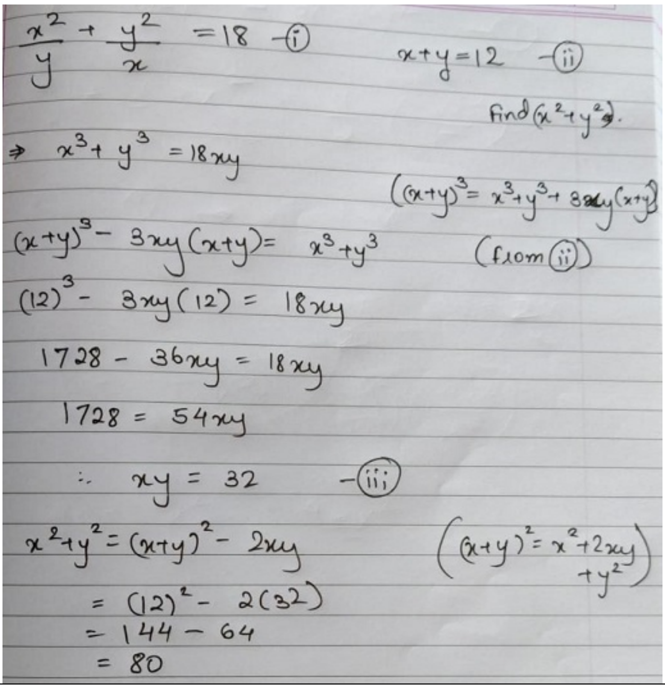


$L^2 = \left(\frac{5\sqrt{2}}{\sqrt{3}-1}\right)^2 = \frac{250}{4-2\sqrt{3}} = \frac{25}{2-\sqrt{3}}$

$\therefore 3L^2 = [PQRS] = \frac{75}{2-\sqrt{3}}$

$\therefore x = 75$

28



$\frac{x^2}{y} + \frac{y^2}{x} = 18$ — ⓘ

$x+y=12$ — ⓘⓘ

Find (x^2+y^2).

$\Rightarrow x^3+y^3 = 18xy$

$((x+y)^3 = x^3+y^3+3xy(x+y))$

$(x+y)^3 - 3xy(x+y) = x^3+y^3$

(from ⓘⓘ)

$(12)^3 - 3xy(12) = 18xy$

$1728 - 36xy = 18xy$

$1728 = 54xy$

$\therefore xy = 32$ — (iii)

$x^2+y^2 = (x+y)^2 - 2xy$

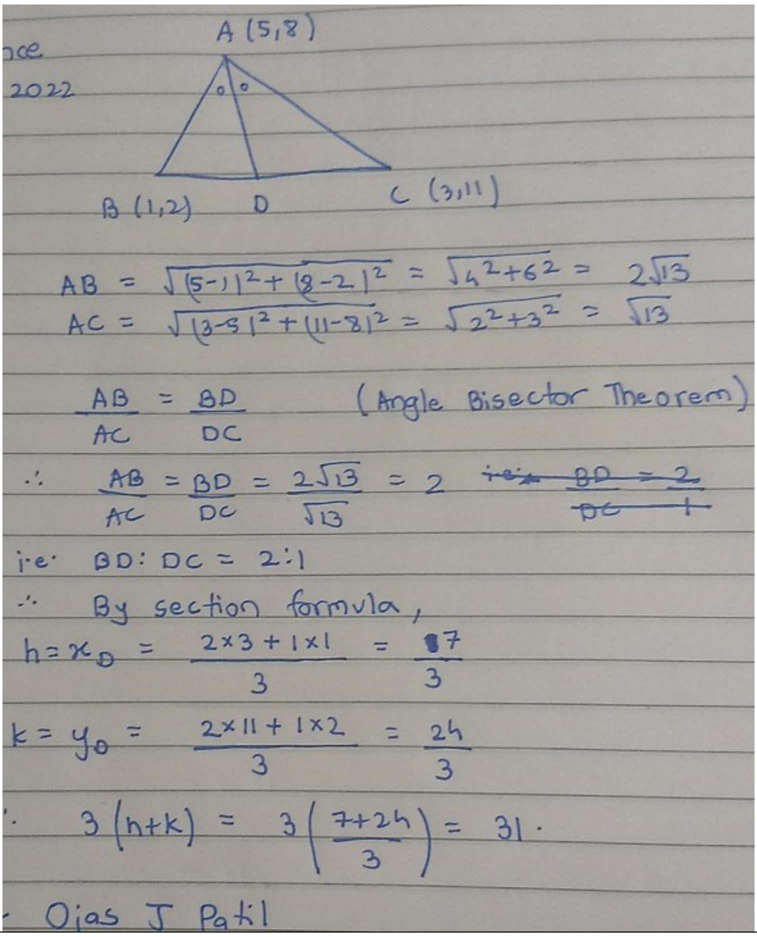
$((x+y)^2 = x^2+2xy+y^2)$

$= (12)^2 - 2(32)$

$= 144 - 64$

$= 80$

29



$AB = \sqrt{(5-1)^2 + (8-2)^2} = \sqrt{4^2+6^2} = 2\sqrt{13}$

$AC = \sqrt{(3-5)^2 + (11-8)^2} = \sqrt{2^2+3^2} = \sqrt{13}$

$\frac{AB}{AC} = \frac{BD}{DC}$ (Angle Bisector Theorem)

$\therefore \frac{AB}{AC} = \frac{BD}{DC} = \frac{2\sqrt{13}}{\sqrt{13}} = 2$ ~~i.e. $\frac{BD}{DC} = \frac{2}{1}$~~

i.e. $BD : DC = 2:1$

$\therefore$ By section formula,

$h = x_D = \frac{2\times 3 + 1\times 1}{3} = \frac{7}{3}$

$k = y_D = \frac{2\times 11 + 1\times 2}{3} = \frac{24}{3}$

$\therefore 3(h+k) = 3\left(\frac{7+24}{3}\right) = 31.$

- Ojas J Patil

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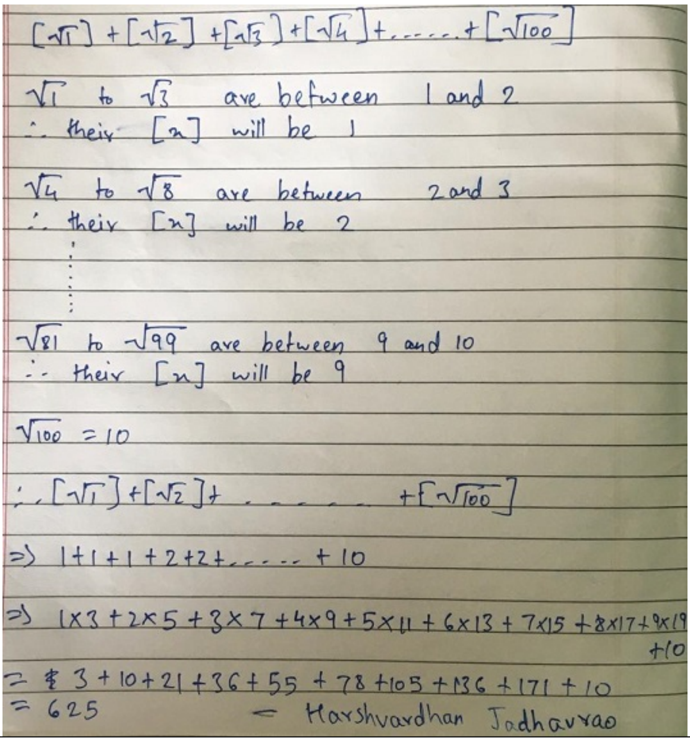

$[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + [\sqrt{4}] + \ldots\ldots + [\sqrt{100}]$

$\sqrt{1}$ to $\sqrt{3}$ are between 1 and 2
$\therefore$ their $[x]$ will be 1

$\sqrt{4}$ to $\sqrt{8}$ are between 2 and 3
$\therefore$ their $[x]$ will be 2

⋮

$\sqrt{81}$ to $\sqrt{99}$ are between 9 and 10
$\therefore$ their $[x]$ will be 9

$\sqrt{100} = 10$

$\therefore [\sqrt{1}] + [\sqrt{2}] + \ldots\ldots + [\sqrt{100}]$

$\Rightarrow 1+1+1+2+2+\ldots\ldots+10$

$\Rightarrow 1\times 3 + 2\times 5 + 3\times 7 + 4\times 9 + 5\times 11 + 6\times 13 + 7\times 15 + 8\times 17 + 9\times 19 + 10$

$= 3 + 10 + 21 + 36 + 55 + 78 + 105 + 136 + 171 + 10$

$= 625$ — Harshvardhan Jadhavrao