M Prakash Institute **XI Entrance Test 2**
Each question carries five marks **10 am to 1 pm** **Paper Type AD**
Student Receipt number: **Student Name:**

Chemistry

Given:
Atomic numbers:

$H=1, B=5, C=6, N=7, O=8, Na=11, Mg=12, Al=13, P=15, S=16, Cl=17,$
$K=19, Ca=20, Sc=21, Ti=22, Mn=25, Fe=26, Cu=29, Zn=30, Br=35,$
$Se=34, Sn=50, Te=52, Ta=73, W=74, Re=75$

Atomic mass:

$H=1, B=11, C=12, N=14, O=16, Na=23, Mg=24, Al=27, P=31, S=32$
$Cl=35.5, K=39, Ca=40, Sc=45, Ti=48, Mn=55, Fe=56, Cu=63.5, Zn=65,$
$Se=79, Br=80, Sn=118, Te=127, Ta=181, W=184, Re=186$

Ques.	1	2	3	4	5	6	7	8	9	10
Ans.	10	38	79	10	12	41	56	14	60	53
Ques.	11	12	13	14	15	16	17	18	19	20
Ans.	10	20	35	65	11	75	3	44	45	7
Ques.	21	22	23	24	25	26	27	28	29	30
Ans.	53	15	6	15	50	22	18	14	5	44

Q1. Elements of fourth period A, B and C belong to the group number third, eleventh and seventeenth of the modern periodic table respectively. The sum of the electrons present in the outermost shell of the elements A, B and C is

Solution: $2+1+7=10$(Elements are Sc, Cu and Br). **Ans. 10.**

Q2. Dobereiner's triad elements X, Y and Z belong to 16^{th} group of the modern periodic table. The sum of atomic number and period number of the middle element is

Solution: $34+4=38$(Elements are S, Se and Te). **Ans. 38.**

Q3.

$$KMnO_4 + FeSO_4 + H_2SO_4 \rightarrow\ K_2SO_4 + MnSO_4 + Fe_2\left(SO_4\right)_3 + H_2O$$

The amount of $KMnO_4$ required to oxidize 2.5 moles of $FeSO_4$ in dilute sulphuric acid medium is ____ gm.

Solution:

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 \rightarrow\ K_2SO_4 + 2MnSO_4 + 5Fe_2\left(SO_4\right)_3 + 8H_2O$$

Since, 10 mol of $FeSO_4$ is oxidized by 2 mol of $KMnO_4$ Hence, 2.5 mol of $FeSO_4$ will be oxidized by 0.5 mol of $KMnO_4$ 0.5 mol of $KMnO_4 = 158\times 0.5 = 79gm$. **Ans. 79.**

Q4. How many from the followings are redox reactions:

$CaCO_{3(s)} \xrightarrow{1000^\circ C} CaO_{(s)} + CO_{2(g)}$	$2H_2O_{2(l)} \rightarrow 2H_2O_{(l)} + O_{2(g)}$
$CuO_{(s)} + H_{2(g)} \rightarrow Cu_{(s)} + H_2O_{(l)}$	$2KMnO_{4(s)} \xrightarrow{\text{Heat}} K_2MnO_{4(s)} + MnO_{2(s)} + O_{2(g)}$
$NH_4NO_{3(s)} \xrightarrow{\text{Heat}} N_2O_{(g)} + H_2O_{(l)}$	$MgH_{2(s)} \xrightarrow{\text{Heat}} Mg_{(s)} + H_{2(g)}$
$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$	$2Mg_{(s)} + O_{2(g)} \rightarrow 2MgO_{(s)}$
$NH_4NO_{2(s)} \xrightarrow{\text{Heat}} N_{2(g)} + H_2O_{(l)}$	$MnO_{2(s)} + 4HCl_{(aq)} \rightarrow MnCl_{2(aq)} + H_2O_{(l)} + Cl_{2(g)}$
$CaO_{(s)} + H_2O_{(l)} \rightarrow Ca(OH)_{2(aq)}$	$CuSO_{4(aq)} + Zn_{(s)} \rightarrow ZnSO_{4(aq)} + Cu_{(s)}$
$NH_{3(g)} + HCl_{(g)} \rightarrow NH_4Cl_{(s)}$	$BaCl_{2(aq)} + Na_2SO_{4(aq)} \rightarrow BaSO_{4(s)} + 2NaCl_{(aq)}$

Solution: Redox reactions are: b, c, d, e, f, g, h, i, j and l **Ans. 10.**

Q5. Cassiterite is a tin ore. It contains mainly the nonmagnetic ingredient SnO_2 and the magnetic ingredient $FeZO_4$. Identify the element ' Z '. The sum of group number and period number of element ' Z ' in the modern periodic table is

Solution: $6+6=12$ (The element is Tungsten W) . **Ans. 12.**

Q6. The amount of water-soluble sodium aluminate formed when 0.25 mole of pure bauxite is treated with aqueous solution of sodium hydroxide is ____ gm.

Solution:

$$Al_2O_3 \cdot 2H_2O_{(s)} + 2NaOH_{(aq)} \rightarrow 2NaAlO_{2(aq)} + 3H_2O_{(l)}$$

Since, one mole of pure bauxite produces 2 mol of sodium aluminate Hence, 0.25 mole of pure bauxite will produce 0.5 mol of sodium aluminate 0.5 mol of sodium aluminate $= 0.5 \times 82 = 41gm$. **Ans. 41.**

Q7. When 397.5 gm of cuprous sulphide is treated with sufficient amount of oxygen to produce copper metal, then the volume of sulphur dioxide gas produced at STP is ____ lit.

Solution: $3Cu_2\ S_{(s)} + 3O_{2(g)} \rightarrow 6Cu_{(s)} + 3SO_{2(g)}$ Since, $477gm(3\ mol)$ of $Cu_2\ S$ gives 3 mol of $SO_{2(g)}$ Hence, $397.5gm(2.5\ mol)$ of Cu_2S will give 2.5 mol of $SO_{2(\ g)}$ 2.5 mol of $SO_{2(g)}$ at $STP = 2.5 \times 22.4 = 56$ lit. **Ans. 56.**

Q8. $CH_3COOH, NH_3, H_2CO_3, KOH, H_3PO_4, Ca(OH)_2, H_3BO_3, H_2SO_4, HNO_3$ The sum of maximum numbers of H^+ions & OH^-ions produced by given acids and bases in water is

Solution: $1+1+2+1+3+2+1+2+1=14$. **Ans. 14.**

Q9. The minimum molecular mass of smallest organic ester is ____ amu.

Solution: The smallest organic ester is Methylformate (Methyl methanoate) $HCOOCH_3$.
Ans. 60.

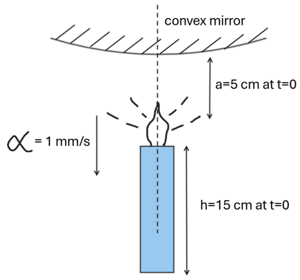
Q10. The molecular mass of monomer unit used for the preparation of polyacrylonitrile polymer is ____ amu.

Solution: The monomer unit used for the preparation of polyacrylonitrile polymer is acrylonitrile $CH_2 = CH - C \equiv N$. **Ans. 53.**

Physics

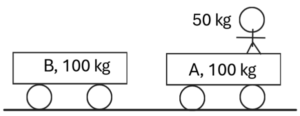
Use $g = 10\ m/s^2$ wherever required.

Q11. A candle of height $h = 15$ cm is placed below a convex mirror. The candle is oriented along the principle axis of the mirror as shown in the figure. The tip of the candle is at the distance of $a = 5$ cm from the pole of the mirror. Candle is lighted at $t = 0$. The wax is consumed such that the flame starts descending at the constant rate of $\alpha = 1$ mm/s. Treat the flame as a point object. At $t = 50$ second the distance between the flame and its image in the mirror is $d = 15$ cm. Determine the value of the focal length of the mirror.



Solution: At time t, the flame is at a distance $u = a + \alpha t$ from the pole of the mirror (it descends away from the mirror as the candle burns down). At $t = 50$ s, $u = 5 + (0.1)(50) = 10$ cm. For a convex mirror, a real object at distance u forms a virtual image (behind the mirror) at distance $v = \dfrac{uf}{u+f}$, and since the image lies behind the mirror while the object is in front of it, the distance between them is $u + v = 15 \Rightarrow v = 5$ cm. So $\dfrac{10f}{10+f} = 5 \Rightarrow 10f = 50 + 5f \Rightarrow f = 10$ cm. **Ans. 10.**

Q12. Figure shows two trolleys A and B each of mass $M = 100$ kg. They are at rest on the ground. Their wheels (of negligible mass) ensure that they can move with negligible friction on the ground. A woman of mass $m = 50$ kg is standing at the right edge of trolley A. She begins to run with speed $u = 90$ cm/s relative to the trolley and after a short time jumps onto trolley B. She comes to rest on trolley B and both move with speed v towards left. Determine value of v in cm/s. When the woman jumps between the trolleys, ignore her downward motion.

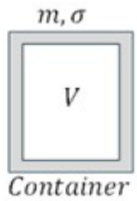


Solution: Take the direction towards B as positive. While the woman (mass m) runs on trolley A (mass M), momentum conservation for (woman+A), starting from rest, gives $m(V_A + u) + MV_A = 0 \Rightarrow V_A = -\dfrac{mu}{m+M} = -\dfrac{u}{3}$ (trolley A recoils away from B). So the woman's speed relative to ground is $V_w = V_A + u = \dfrac{2u}{3}$ towards B. When she lands on and comes to rest relative to trolley B, momentum conservation for (woman+B) gives $mV_w = (m+M)v \Rightarrow v = \dfrac{m}{m+M} \cdot \dfrac{2u}{3} = \dfrac{50}{150} \times \dfrac{2(90)}{3} = 20$ cm/s. **Ans. 20.**

Q13. A body is thrown upwards from the ground. Consider the situation in which the resistance offered by the air is not negligible. As a matter of fact, assume that this resistance, expressed as a force, is constant and is expressed as $F = \dfrac{W}{\alpha}$. Here W is the weight of the body and α is some fraction. The resistive force always acts in the direction opposite to the velocity of the body. It is observed that the time it takes the body to fall back to the ground is twice the time it takes to reach a maximum height. If $\alpha = \dfrac{N}{21}$, calculate the value of N.

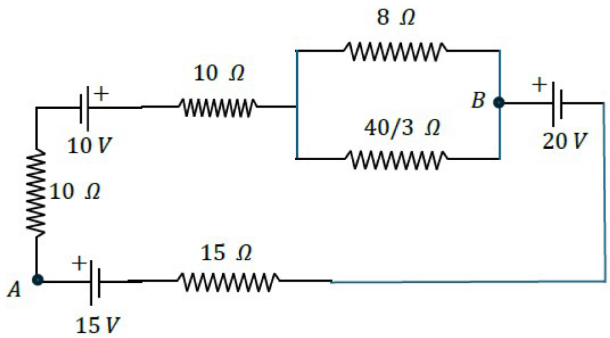
Solution: While going up, the retardation is $g\left(1 + \dfrac{1}{\alpha}\right)$ (gravity and resistance both act downward); while coming down, the acceleration is $g\left(1 - \dfrac{1}{\alpha}\right)$ (resistance now acts upward). Since the maximum height H is the same either way, $H = \dfrac{1}{2}g\left(1 + \dfrac{1}{\alpha}\right)t_1^2 = \dfrac{1}{2}g\left(1 - \dfrac{1}{\alpha}\right)t_2^2$. Using $t_2 = 2t_1$, $1 + \dfrac{1}{\alpha} = 4\left(1 - \dfrac{1}{\alpha}\right) \Rightarrow \dfrac{5}{\alpha} = 3 \Rightarrow \alpha = \dfrac{5}{3} = \dfrac{N}{21} \Rightarrow N = 35$. **Ans. 35.**

Q14. An empty closed container has mass $m = 100$ gram and is made of material of density $\sigma = \dfrac{10}{23}$ gram/cc. Space enclosed inside the container has volume $V = 100$ cm^3. See figure. The container is now partially filled with water having volume $V' < V$. When the container is placed in a large body of water, it is found to be half submerged. Determine value of V', the volume of water inside the container (not shown in the figure). Express your answer in cm^3. Take density of water as 1 gram/cc.



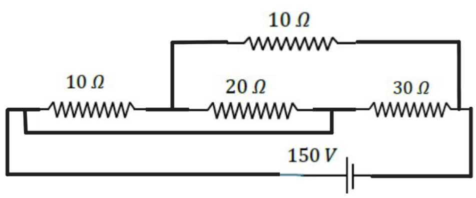
Solution: Volume of the solid material of the container itself is $\dfrac{m}{\sigma} = \dfrac{100}{10/23} = 230$ cm^3. Since the container is closed, its total outer (displacing) volume is (material)+(sealed cavity)= $230 + 100 = 330$ cm^3. Half submerged $\Rightarrow$ volume of water displaced $= 165$ cm^3, i.e. buoyant force $= 165$ gm-wt. Equating this to the total weight of the container (including the water put inside it), $100 + V' = 165 \Rightarrow V' = 65$ cm^3. **Ans. 65.**

Q15. In the following circuit a steady current is flowing in all the branches. Let the magnitude of the potential difference between points A and B be V_{AB}. Now assume that all the batteries are shorted, that is, they are removed and replaced by conducting wires in their place. Then let the equivalent resistance between the points A and B be equal to R_{AB}. Let the ratio $\dfrac{V_{AB}}{R_{AB}}$ be equal to $\dfrac{N}{15}$ Ampere. Determine value of N.

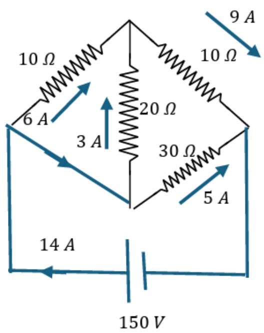


Solution: The 8 Ω and 40/3 Ω resistors combine to 5 Ω, so the whole network is a single loop carrying one current I. Applying Kirchhoff's voltage law around the loop (with the battery polarities shown), the net EMF is $10-20+15 = 5$ V and the total resistance is $10+10+5+15 = 40$ Ω, so $I = \dfrac{5}{40} = \dfrac{1}{8}$ A. Taking $V_A = 0$ and tracking the potential around the loop through this current gives $V_B = \dfrac{55}{8}$ V, so $V_{AB} = \dfrac{55}{8}$ V. Shorting all three batteries, A and B are joined by two resistive paths in parallel: $10 + 10 + 5 = 25$ Ω and 15 Ω, so $R_{AB} = \dfrac{25 \times 15}{25 + 15} = \dfrac{75}{8}$ Ω. Hence $\dfrac{V_{AB}}{R_{AB}} = \dfrac{55/8}{75/8} = \dfrac{11}{15} \Rightarrow N = 11$. **Ans. 11.**

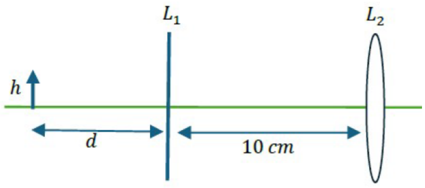
Q16. The following circuit has steady current flowing through all the branches. Let the maximum power dissipated in any resistor be expressed as $P = 10 \times N$ [Watt]. Determine the value of N.



Solution: Redraw the circuit as shown. Identifying series and parallel combinations, we get current distribution as shown. Using $P = i^2R$, maximum power flows through right side 10 Ohm resistor, equal to 810 Watt. **Ans. 81.**

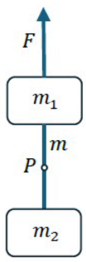


Q17. Consider lens L_1 which may be convex or concave. It's nature is such that a real object of height $h = 5$ mm placed to its left side, at the distance of $d = 5$ cm forms an image that is erect, virtual and having a size of $h' = 4$ mm. Lens L_2 is a convex lens. Magnitude of its focal length is $\frac{3}{10}$ times the magnitude of focal length of lens L_1. Distance between L_1 and L_2 is 10 cm. Determine height h'' of the final image formed. Express your answer in mm.



Solution: Since the image formed by L_1 is virtual, erect and smaller than the object, L_1 must be concave (diverging). Its magnification is $m_1 = \frac{h'}{h} = \frac{4}{5}$. With $u = -5$ cm, $v = m_1 u = -4$ cm, so $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow -\frac{1}{4} + \frac{1}{5} = \frac{1}{f_1} \Rightarrow f_1 = -20$ cm. So $|f_1| = 20$ cm and $f_2 = \frac{3}{10}(20) = 6$ cm (convex). The virtual image from L_1 lies 4 cm to the left of L_1, i.e. $10 + 4 = 14$ cm to the left of L_2, so for L_2, $u_2 = -14$ cm. Then $\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{6} - \frac{1}{14} = \frac{2}{21} \Rightarrow v_2 = \frac{21}{2}$ cm, and $m_2 = \frac{v_2}{u_2} = -\frac{3}{4}$. The overall magnification is $m_1 m_2 = \frac{4}{5} \times \left(-\frac{3}{4}\right) = -\frac{3}{5}$, so $h'' = \left|-\frac{3}{5}\right| \times 5 = 3$ mm. **Ans. 03.**

Q18. Two blocks of masses $m_1 = 5$ kg and $m_2 = 3$ kg are connected by a heavy and inextensible rope of mass $m = 2$ kg. The system of blocks and the rope is pulled up with a force $F = 110$ N. Determine tension T at the midpoint P of the rope. Take $g = 10$ m/s^2.



Solution: Total mass of the system is $m_1 + m + m_2 = 10$ kg, with weight 100 N. Net force $= F - 100 = 10$ N gives acceleration $a = 1$ m/s^2 upward. Now consider only the part of the system below P, namely m_2 together with the lower half of the rope: mass $= m_2 + \frac{m}{2} = 3 + 1 = 4$ kg. For this part, $T - 40 = 4a = 4 \Rightarrow T = 44$ N. **Ans. 44.**

Q19. Assume a straight road connecting two cities A and B. Two buses left city A at an interval of 10 minute and they travel towards city B. Speed of both buses is constant and equals 30 km/h. A bus going from city B towards city A passed these two buses at an interval of 4 minute. Determine the speed of the bus going from B to A. Assume this speed to be constant. Express your answer in km/h.

Solution: Successive buses leaving A (both at 30 km/h) are always $30 \times \frac{10}{60} = 5$ km apart. The bus coming from B (speed v) meets them at intervals of 4 min$= \frac{1}{15}$ h, and since the two buses approach each other, the closing (relative) speed is $v + 30$. So $\frac{5}{v+30} = \frac{1}{15} \Rightarrow v + 30 = 75 \Rightarrow v = 45$ km/h. **Ans. 45.**

Q20. 5 gram of ice at -20° C is mixed with 21 gram of water at 30° C in an insulated container which does not absorb any heat. Determine the final temperature of the mixture. Round off your answer to the nearest integer. For example, if the answer is 12.3 write it as 12; if it is 12.5 write it as 12; if it is 12.6 write it as 13.

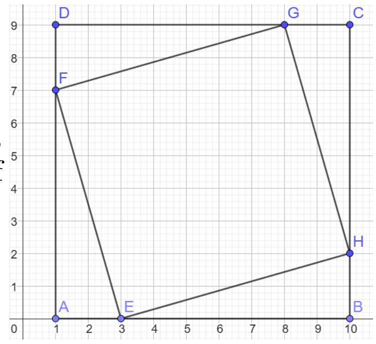
Given: Specific heat capacity of ice: 0.5 cal/(g.$^\circ$ C),
Latent heat of fusion of ice: 80 cal/g

Solution: Heat needed to bring 5 g of ice from -20°C to water at 0°C is $5(0.5)(20) + 5(80) = 50 + 400 = 450$ cal. Heat available on cooling 21 g of water from 30°C to 0°C is $21(1)(30) = 630$ cal.

Since $630 > 450$, all the ice melts, leaving $630 - 450 = 180$ cal to warm the resulting 26 g of water above 0°C: $180 = 26T \Rightarrow T = \dfrac{180}{26} = 6.92$°C, which rounds to 7°C. **Ans. 07.**

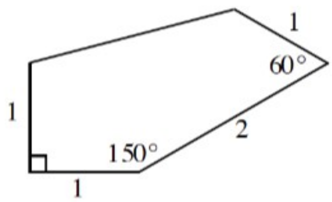
Maths

Q21. The x-coordinates of the vertices of a square in the plane are $1, 3, 8$ and 10. Determine the area of the square.

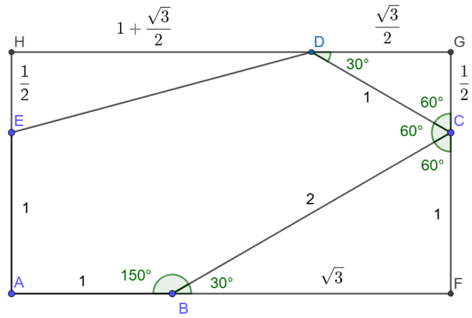


Solution: Refer to the diagram. The four triangles, namely, $\triangle AEF, \triangle BHE, \triangle HCG, \triangle GDF$ are congruent, giving the side of the $\square EFGH = \sqrt{53}$. **Ans. 53.**

Q22. If area of the pentagon shown in the figure is $\frac{a+b\sqrt{c}}{d}$ then $a+b+c+d$ equals.
Note that c is a prime number and a, d and b, d are relatively prime numbers.



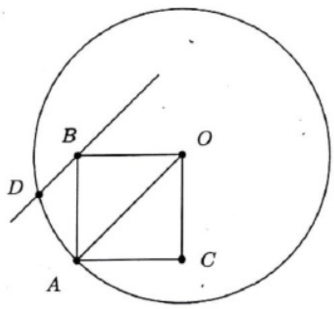
Solution: As shown in the diagram, complete the rectangle $AFGH$. Various sides can be calculated using properties of $30-60-90$ triangle, as shown. So, we have:
area($\square AFGH$) $= (1+\sqrt{3})\frac{3}{2}$ area($\triangle BFC$) $= \frac{\sqrt{3}}{2}$
area($\triangle CGD$) $= \frac{\sqrt{3}}{8}$ area($\triangle DHE$) $= \frac{2+\sqrt{3}}{8}$ $\Rightarrow$
area of pentagon $= (1+\sqrt{3})\frac{3}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8} + \frac{2+\sqrt{3}}{8}\right) = \frac{5+3\sqrt{3}}{4}$. **Ans. 15.**

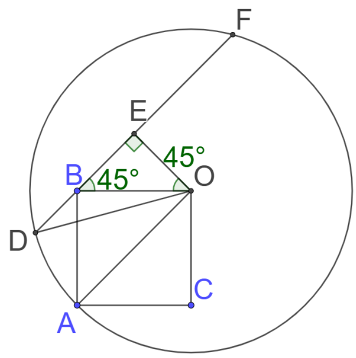


Q23. Natural numbers a, b are such that
(i) the sum of their squares is S
(ii) the sum of their cubes is C times the sum of the numbers
(iii) $S - C = 28$.
The number of such pairs (a, b) is

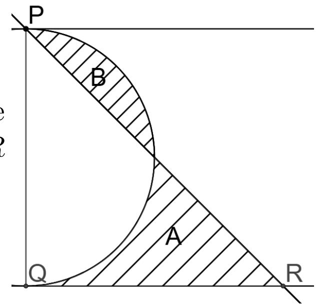
Solution: Since $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$, the condition gives $C = a^2 - ab + b^2$. So $S - C = (a^2+b^2) - (a^2-ab+b^2) = ab = 28$. So, the possible pairs are $(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)$. **Ans. 6.**

Q24. As shown in figure, O is the centre of the circle. $ACOB$ is a square with A on the circle. Through B a line parallel to OA is drawn to cut the circle at D nearer to A. Then $\angle BOD$ equals





Solution: As shown in the figure, draw a $\perp$ from O on line DB to intersect it in E. If $OA = r$ then $OB = \frac{r}{\sqrt{2}}$ and therefore, $OE = \frac{r}{2}$. But $OD = r$, so $\triangle ODE$ is a 30-60-90 triangle with $\angle EDO = 30°$ and $\angle EOD = 60°$. But $\angle BOE = 45°$ so $\angle BOD = 15°$. **Ans. 15.**



Q25. As shown in figure 6, $PQ = 42$. QR is the tangent to the semicircle at Q. If the difference of the areas of regions A, B is 357 , then QR equals.Take $\pi = \frac{22}{7}$.

Solution: Since $\angle Q = 90°$, the semicircle drawn on $PQ = 42$ as diameter (radius 21) is automatically tangent to QR at Q. Its area is $\dfrac{1}{2}\pi(21)^2 = \dfrac{1}{2} \times \dfrac{22}{7} \times 441 = 693$. If Z is the area common to the semicircle and $\triangle PQR$, then region $A = [\triangle PQR] - Z$ and region $B = 693 - Z$, so Area(A)$-$Area(B) $= [\triangle PQR] - 693$. Given this equals 357, $[\triangle PQR] = 1050$. Since $[\triangle PQR] = \dfrac{1}{2}(PQ)(QR) = 21\,QR$, we get $QR = \dfrac{1050}{21} = 50$. **Ans. 50.**

Q26. If $f(x)$ is a polynomial of degree three with leading coefficient 1 such that $f(1) = 1, f(2) = 4, f(3) = 9$, then the value of $f(4)$ is

Solution: Let $g(x) = f(x) - x^2$. Since f is monic cubic, so is g, and $g(1) = g(2) = g(3) = 0$ (as $f(1) = 1^2, f(2) = 2^2, f(3) = 3^2$), so $g(x) = (x-1)(x-2)(x-3)$. Hence $f(4) = 4^2 + (3)(2)(1) = 16 + 6 = 22$. **Ans. 22.**

Q27. Given

$$\begin{aligned} a &= -\sqrt{99} + \sqrt{999} + \sqrt{9999} \\ b &= \sqrt{99} - \sqrt{999} + \sqrt{9999} \\ c &= \sqrt{99} + \sqrt{999} - \sqrt{9999} \end{aligned}$$

then find the sum of the digits of

$$\frac{a^4}{(a-b)(a-c)} + \frac{b^4}{(b-c)(b-a)} + \frac{c^4}{(c-a)(c-b)}$$

Solution: Let $x = \sqrt{99}, y = \sqrt{999}, z = \sqrt{9999}$, so $a = y + z - x,\ b = z + x - y,\ c = x + y - z$.
$\dfrac{a^4}{(a-b)(a-c)} + \dfrac{b^4}{(b-c)(b-a)} + \dfrac{c^4}{(c-a)(c-b)} = a^2 + b^2 + c^2 + ab + bc + ca$
$= \frac{1}{2}((a+b)^2 + (b+c)^2 + (c+a)^2)$
$a + b = 2z,\ b + c = 2x,\ c + a = 2y \Rightarrow \frac{1}{2}((a+b)^2 + (b+c)^2 + (c+a)^2) = 2(x^2 + y^2 + z^2) = 2(99 + 999 + 9999) = 22194$. The digit sum of 22194 is $2 + 2 + 1 + 9 + 4 = 18$. **Ans. 18.**

Q28. The polynomial $g(x) = x^3 + ax^2 + x + 10$ has three distinct roots and each root of $g(x) = 0$ is also a root of $f(x) = x^4 + x^3 + bx^2 + 100x + c$. Find sum of the digits of $|f(1)|$ that is absolute value of $f(1)$.

Solution: Since all three (distinct) roots of g are also roots of f, g divides f, and comparing degrees, $f(x) = (x-r)(x^3+ax^2+x+10)$ for some real r. Expanding and comparing with $f(x) = x^4+x^3+bx^2+100x+c$: the coefficient of x gives $10-r=100 \Rightarrow r=-90$; the coefficient of x^3 gives $a-r=1 \Rightarrow a=-89$; the coefficient of x^2 gives $b = 1-ar = 1-(-89)(-90) = 1-8010 = -8009$; the constant term gives $c = -10r = 900$. So $f(1) = 1+1+b+100+c = 102+(-8009)+900 = -7007$, and $|f(1)| = 7007$ has digit sum $7+0+0+7 = 14$. **Ans. 14.**

Q29. In an infinite geometric progression with common ratio $r < 1$, every term is 4 times as large as the sum of all its successive terms. The value of $\frac{1}{r}$ is .

Solution: If T is any term, the sum of all its successive terms is $\dfrac{Tr}{1-r}$.
The condition $T = 4 \cdot \dfrac{Tr}{1-r}$ gives $1-r = 4r \Rightarrow r = \dfrac{1}{5} \Rightarrow \dfrac{1}{r} = 5$. **Ans. 5.**

Q30. If $\sec x + \tan x = \frac{22}{7}$ and $\operatorname{cosec} x + \cot x = \frac{m}{n}$, where m, n are integers and have no common factors > 1, the value of $m+n$ is.

Solution: Let $\sec x + \tan x = a \Rightarrow \sec x - \tan x = \dfrac{1}{\sec x + \tan x} = \dfrac{1}{a}$.
Adding the equations, we get $2\sec x = a + \frac{1}{a} = \frac{a^2+1}{a} \Rightarrow \cos x = \frac{2a}{a^2+1}$
Subtracting the equations, we get $2\tan x = a - \frac{1}{a} \Rightarrow \tan x = \frac{a^2-1}{2a}$
$\sin x = \tan x \cos x = \frac{a^2-1}{a^2+1} \Rightarrow \operatorname{cosec} x + \cot x = \frac{a^2+1}{a^2-1} + \frac{2a}{a^2-1} = \frac{(a+1)^2}{a^2-1} = \frac{a+1}{a-1} = \frac{\frac{22}{7}+1}{\frac{22}{7}-1} = \frac{29}{15}$
so $m = 29, n = 15$ (coprime) and $m+n = 44$. **Ans. 44.**